Primordial non-Gaussianity & the Galaxy Bispectrum

Cosmological non-Gaussianity Workshop
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w/ Martin Crocce & Vincent Desjacques

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Matter correlators

a bit of Perturbation Theory ...

$$P = P_0 + P_G^{loop}[P_0] + P_{NG}^{loop}[P_0, B_0]$$

matter power spectrum

Linear power spectrum

Gravity-induced contributions (depending on P_0 alone)

Additional gravity-induced contributions present *only* for NG initial conditions (B_0)

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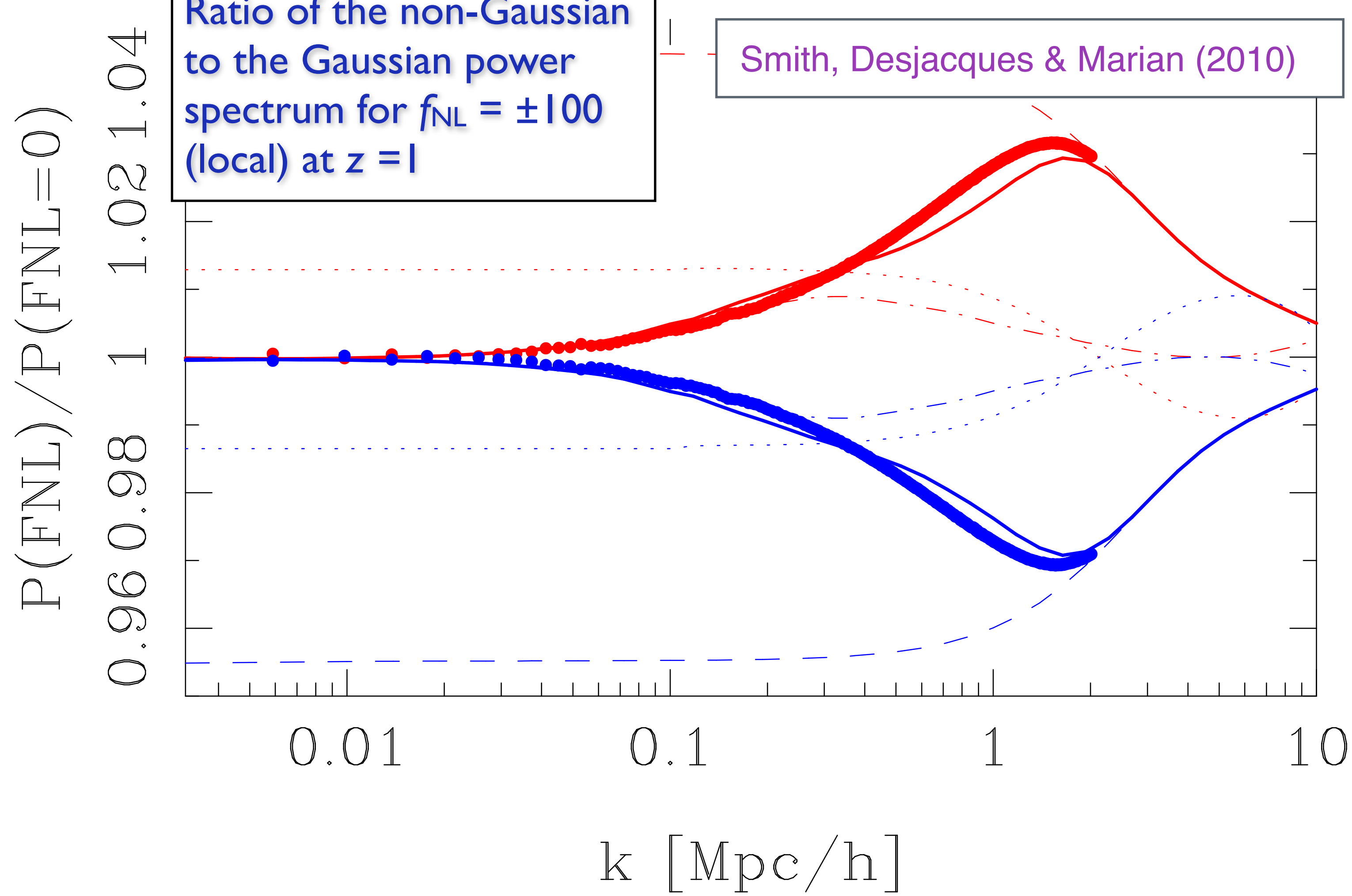
Few percent effect at small scales for allowed values of f_{NL}

Ratio of the non-Gaussian to the Gaussian power spectrum for f_{NL} = ±100 (local) at z =1

Smith, Desjacques & Marian (2010)



Matter correlators

a bit of Perturbation Theory ...

$$P = P_0 + P_G^{loop}[P_0] + P_{NG}^{loop}[P_0, B_0]$$

matter power spectrum

- P_0: Linear power spectrum
- $P_G^{loop}[P_0]$: Gravity-induced contributions (depending on P_0 alone)
- $P_{NG}^{loop}[P_0, B_0]$: **Additional** gravity-induced contributions present *only* for NG initial conditions (B_0)

$$B = B_0 + B_G^{tree}[P_0] + B_G^{loop}[P_0] + B_{NG}^{loop}[P_0, B_0]$$

& **bispectrum**

- B_0: **Primordial component**
- $B_G^{tree}[P_0]$, $B_G^{loop}[P_0]$: Gravity-induced contributions (depending on P_0 alone)
- $B_{NG}^{loop}[P_0, B_0]$: **Additional** gravity-induced contributions present *only* for NG initial conditions (B_0)

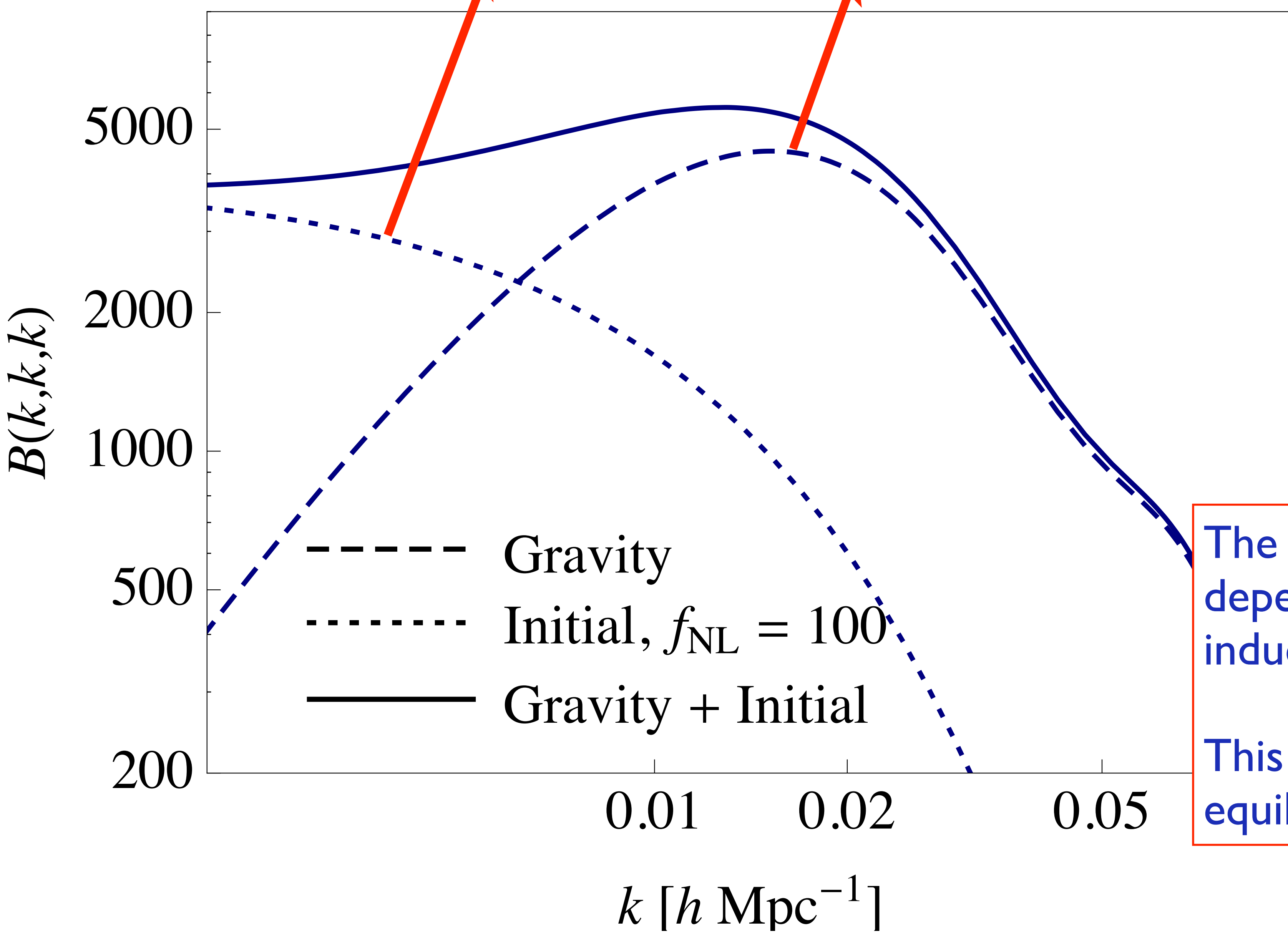
The matter bispectrum and PNG: *large scales*

At large scales

$$B(k_1, k_2, k_3) \simeq B_0 + B_G^{tree}[P_0]$$

Primordial component

Gravity-induced component



Equilateral configurations of the matter bispectrum

$$\frac{B_0(k,k,k)}{B_G^{tree}(k,k,k)} \overset{k\to 0}{\sim} \frac{f_{NL}}{D(z)k^2}$$

The primordial component has a different dependence on scale than the gravity-induced one

This is true for almost all models (local, equilateral, orthogonal ...)

The matter bispectrum and PNG: *large scales*

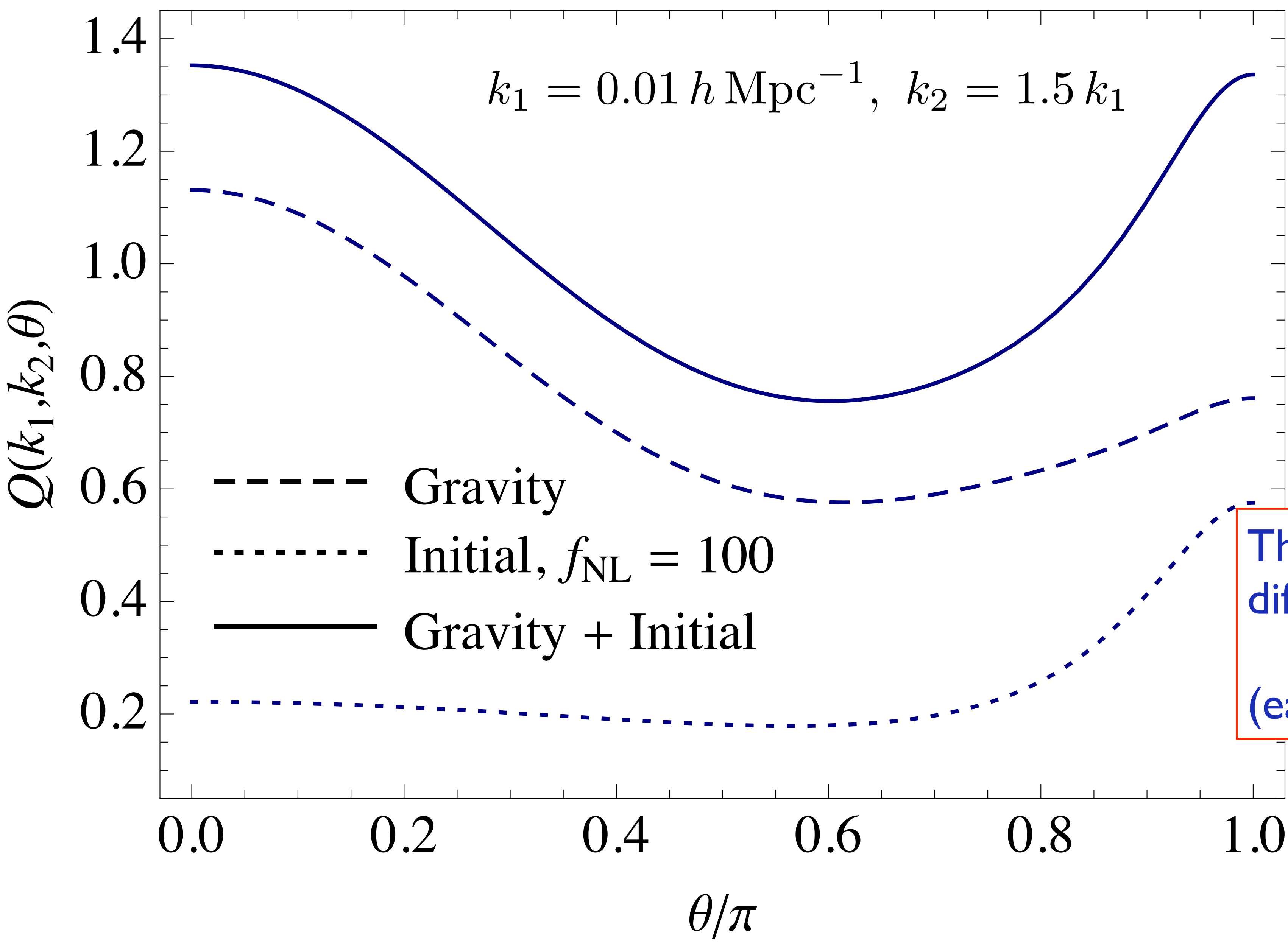
At large scales

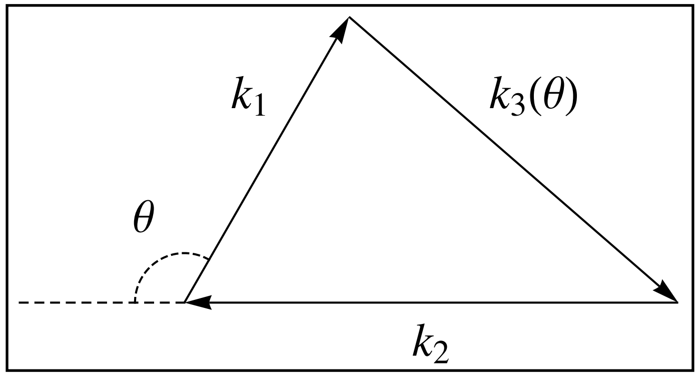
$$B(k_1, k_2, k_3) \simeq B_0 + B_G^{tree}[P_0]$$

Primordial component

Gravity-induced component

$$Q = \frac{B}{P(k_1)P(k_2) + cyc.}$$

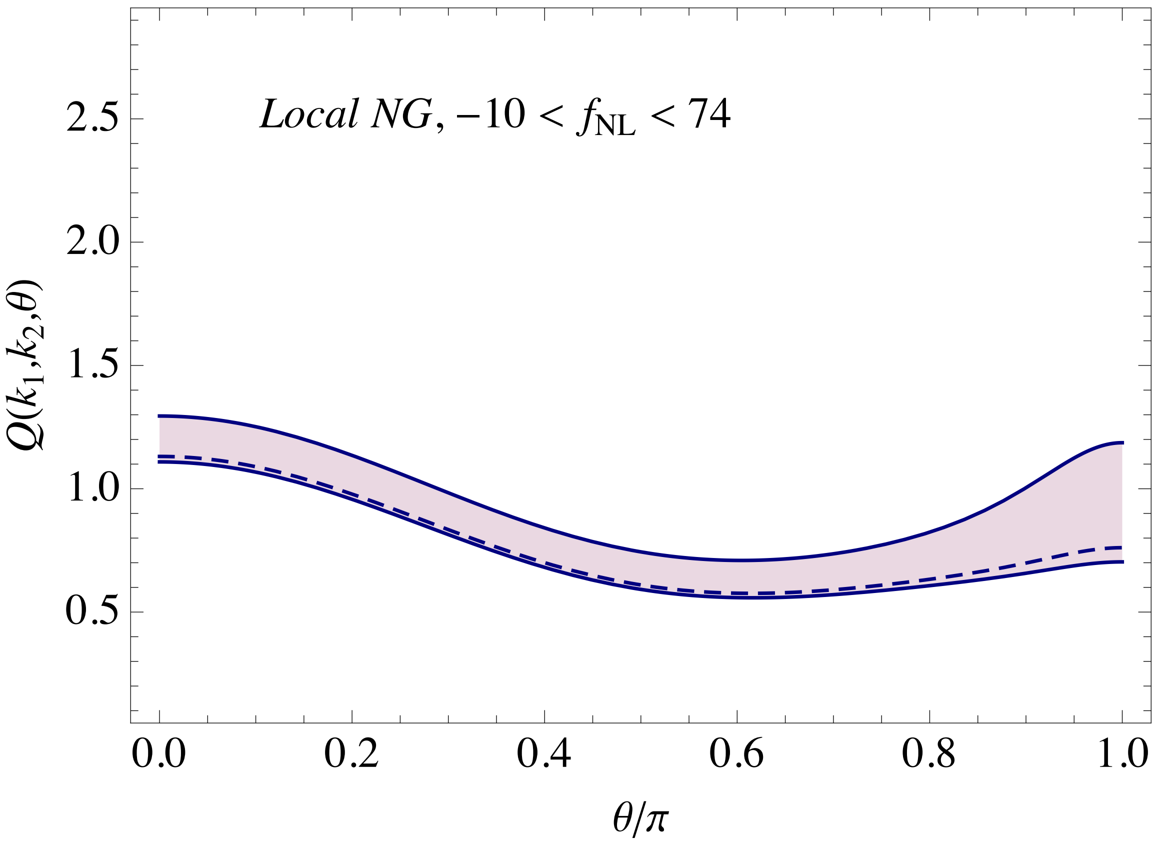




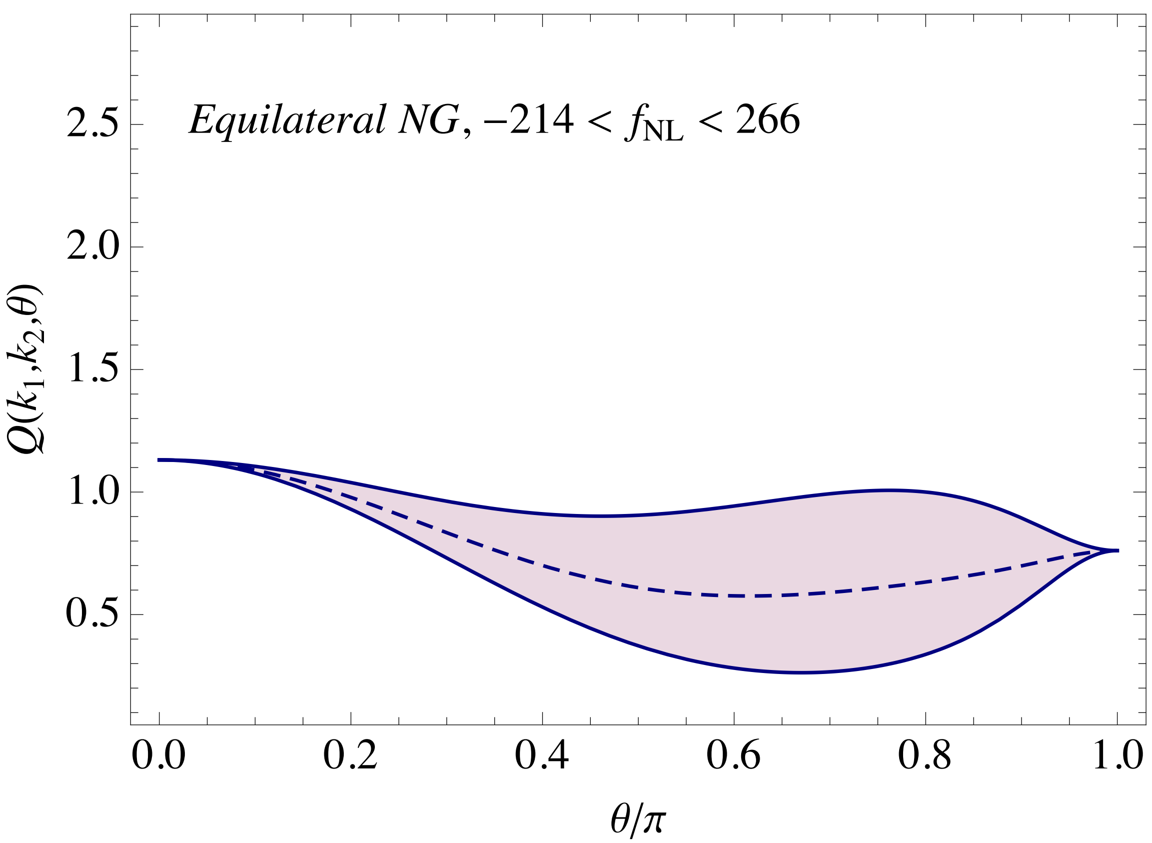
The primordial component has a different shape dependence

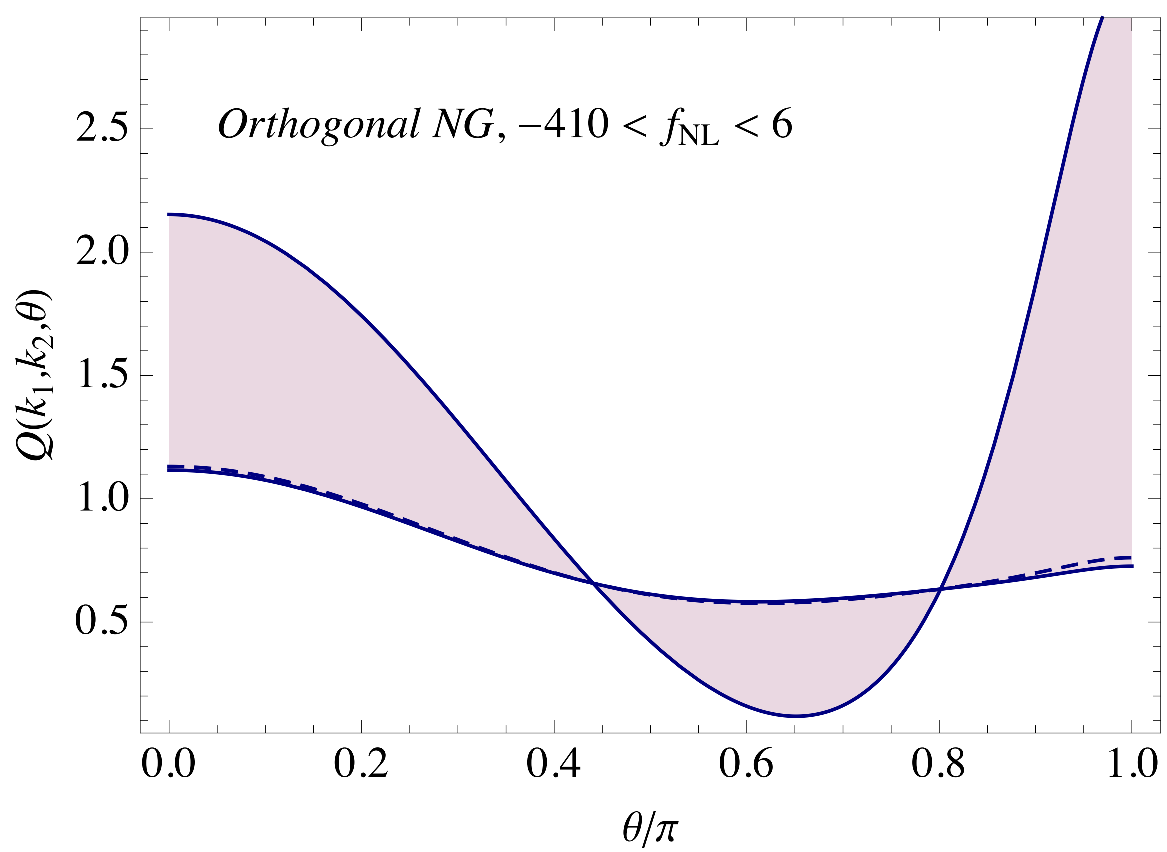
(each model has its own, of course)

The matter bispectrum and PNG: *large scales*



Current CMB constraints for different models of non-Gaussianity as uncertainties on generic configurations of the matter bispectrum, $B \simeq B_0 + B_G^{tree}[P_0]$





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If B_0 was the *only* **effect** of NG initial conditions on the LSS then future, large volume surveys (~100 Gpc^3) could provide:

$$\Delta f_{NL}^{local} < 5 \text{ and } \Delta f_{NL}^{eq} < 10$$

ES & Komatsu (2007)

Matter correlators

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Nonlinear corrections are *also* affected by the initial conditions!

There is a **significant effect** of NG initial conditions of about 5-15% on all triangles, at **small scales** and at **late times** for $f_{NL} = 100$

The matter bispectrum and PNG: *small scales*

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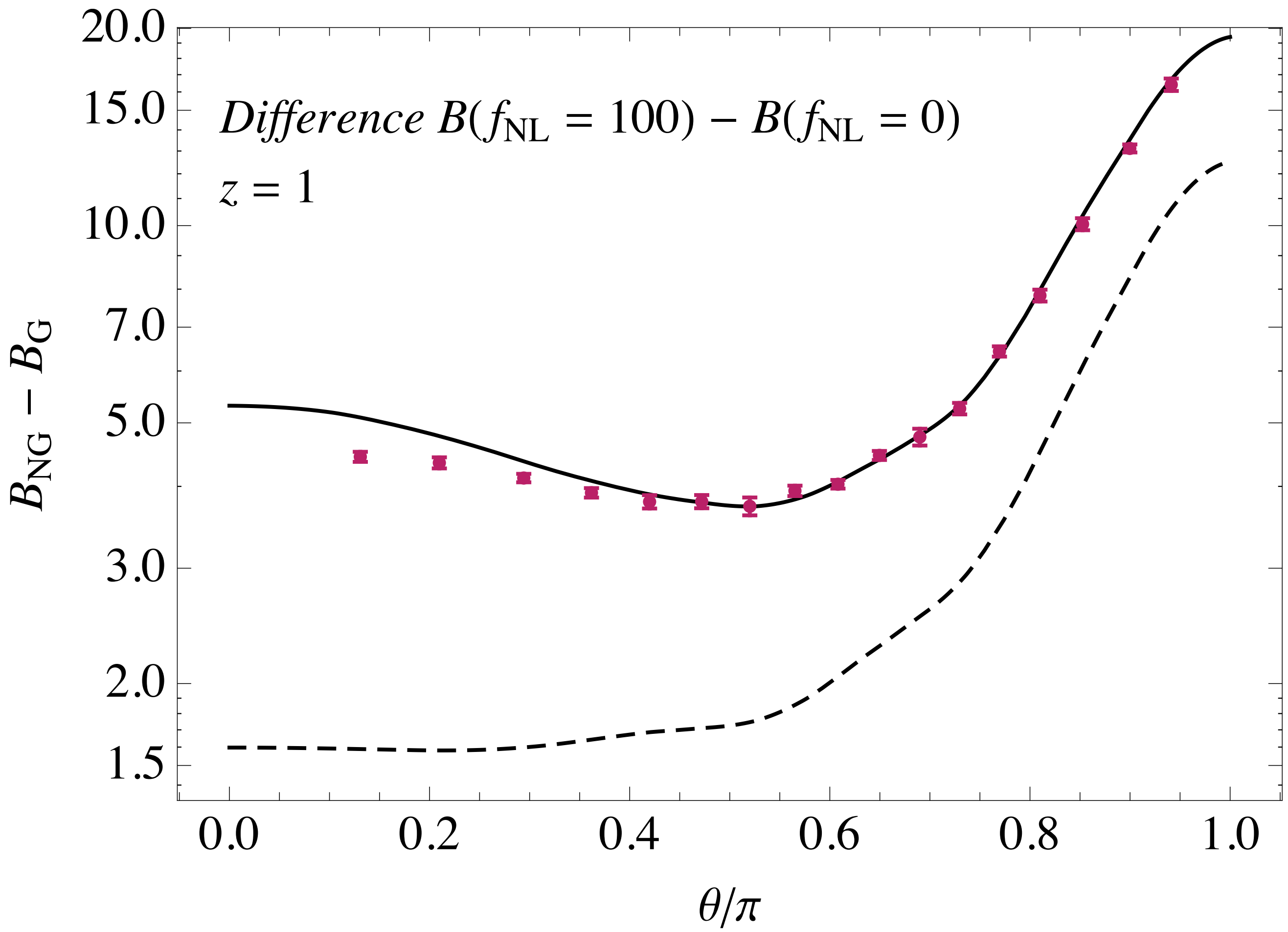
Primordial component

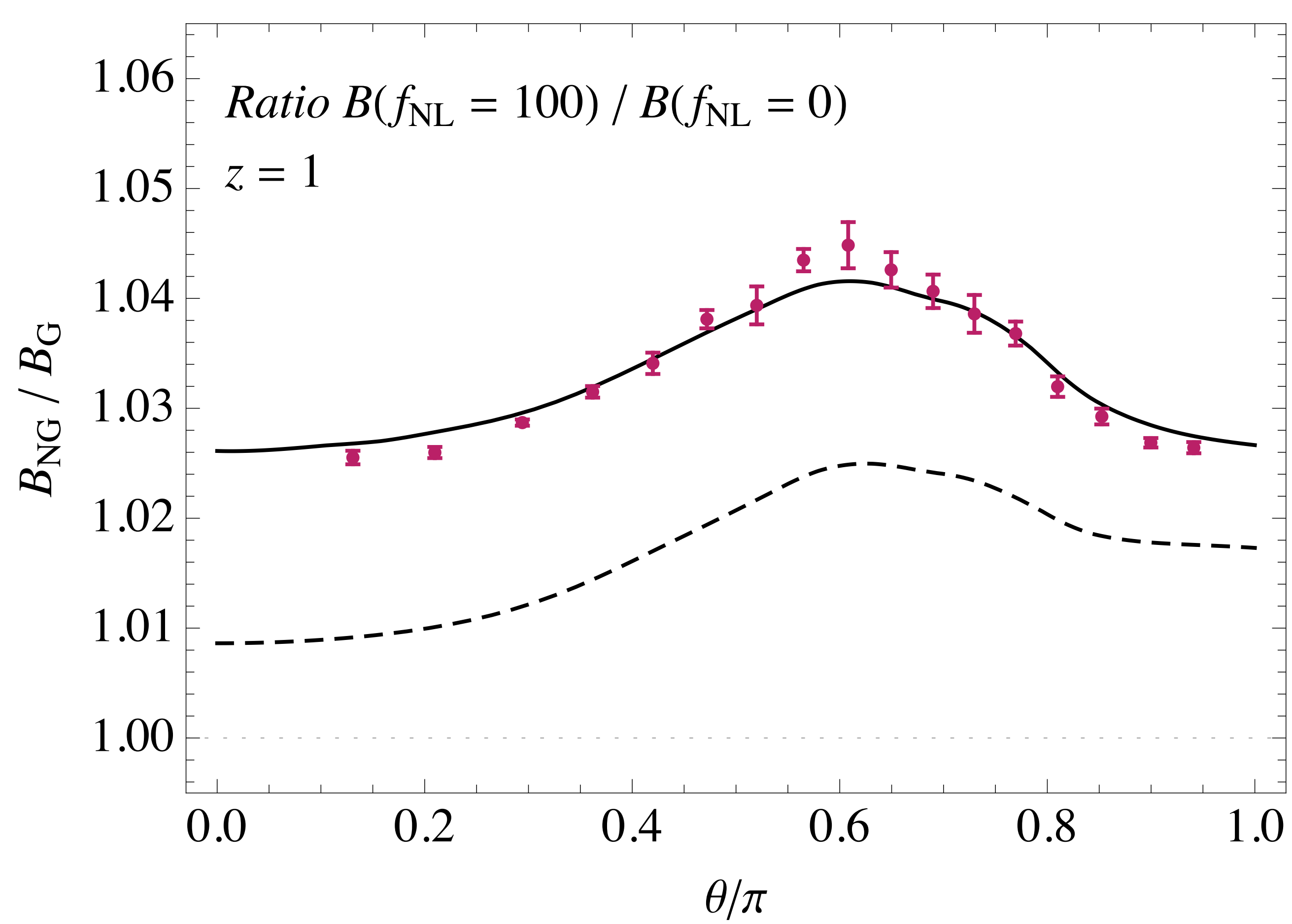
Gravity-induced contributions

Additional gravity-induced contributions present for NG initial conditions (B_0)

Generic configurations $B(k_1, k_2, \theta)$ as a function of θ with k_1 = 0.1 h/Mpc, k_2 = 1.5 k_1

ES (2009)
ES, Crocce & Desjacques (2010)





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Squeezed configurations $B(\Delta k, k, k)$ as a function of k with $\Delta k = 0.01\ h$/Mpc

ES (2009)
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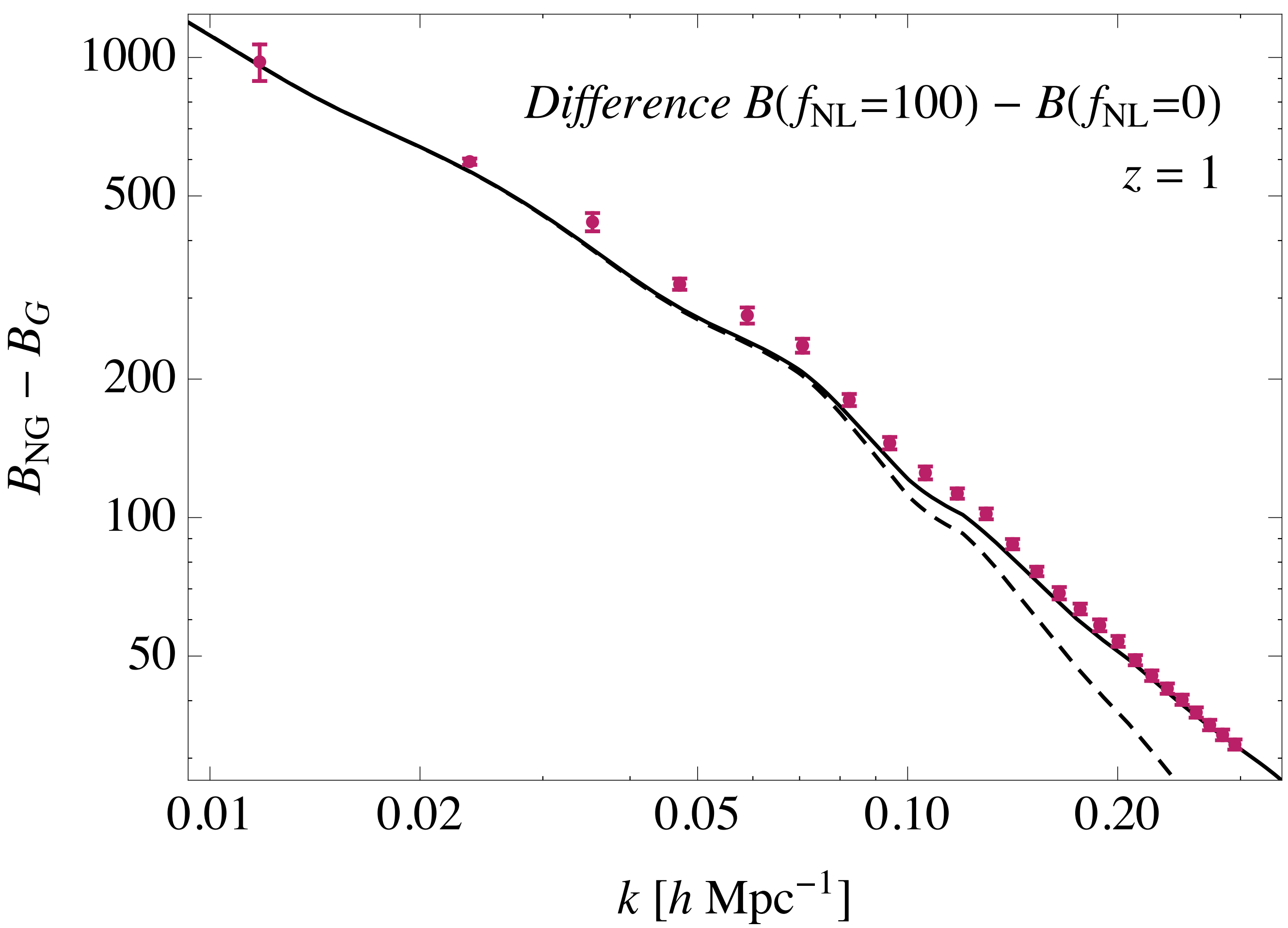
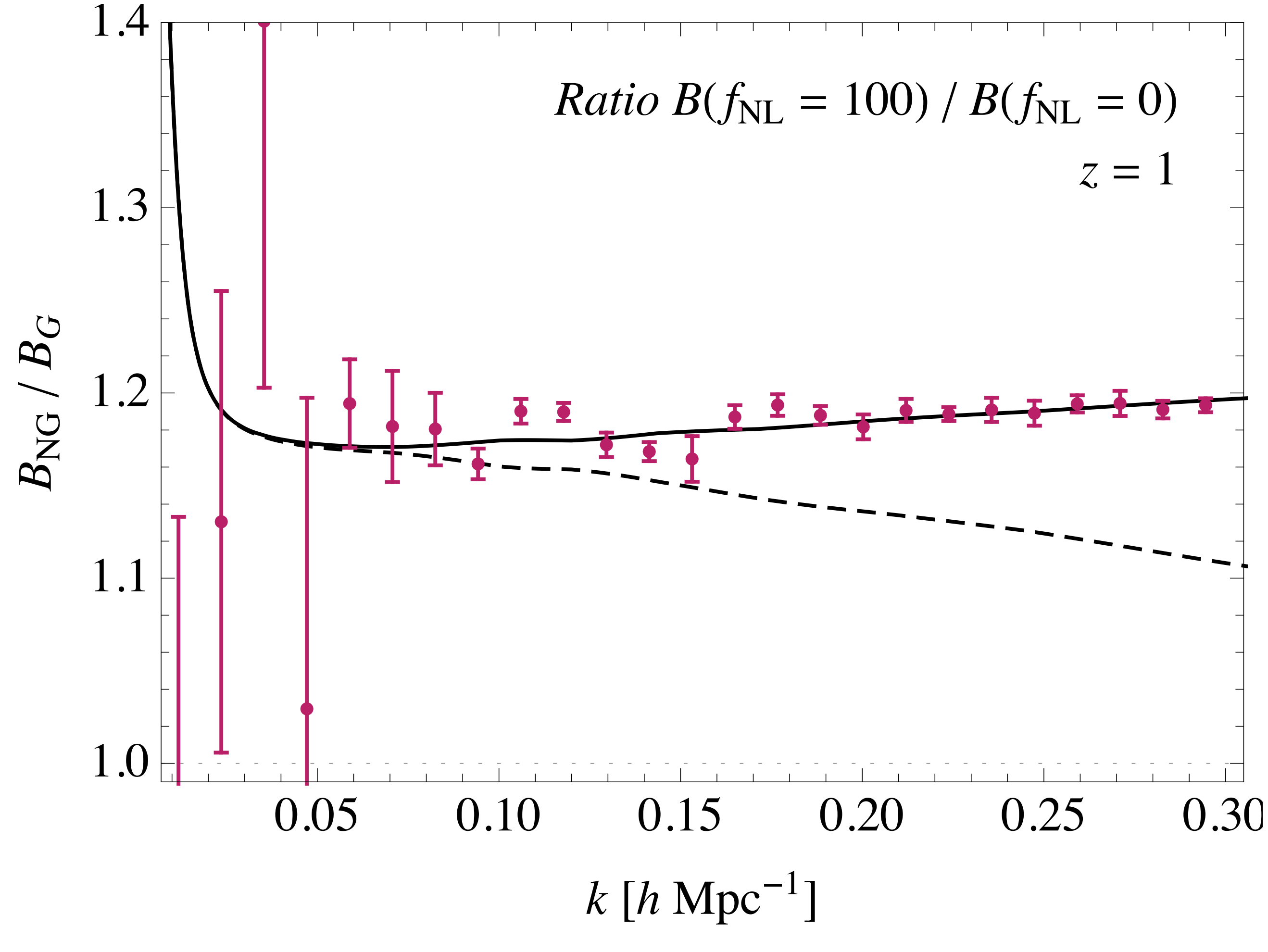
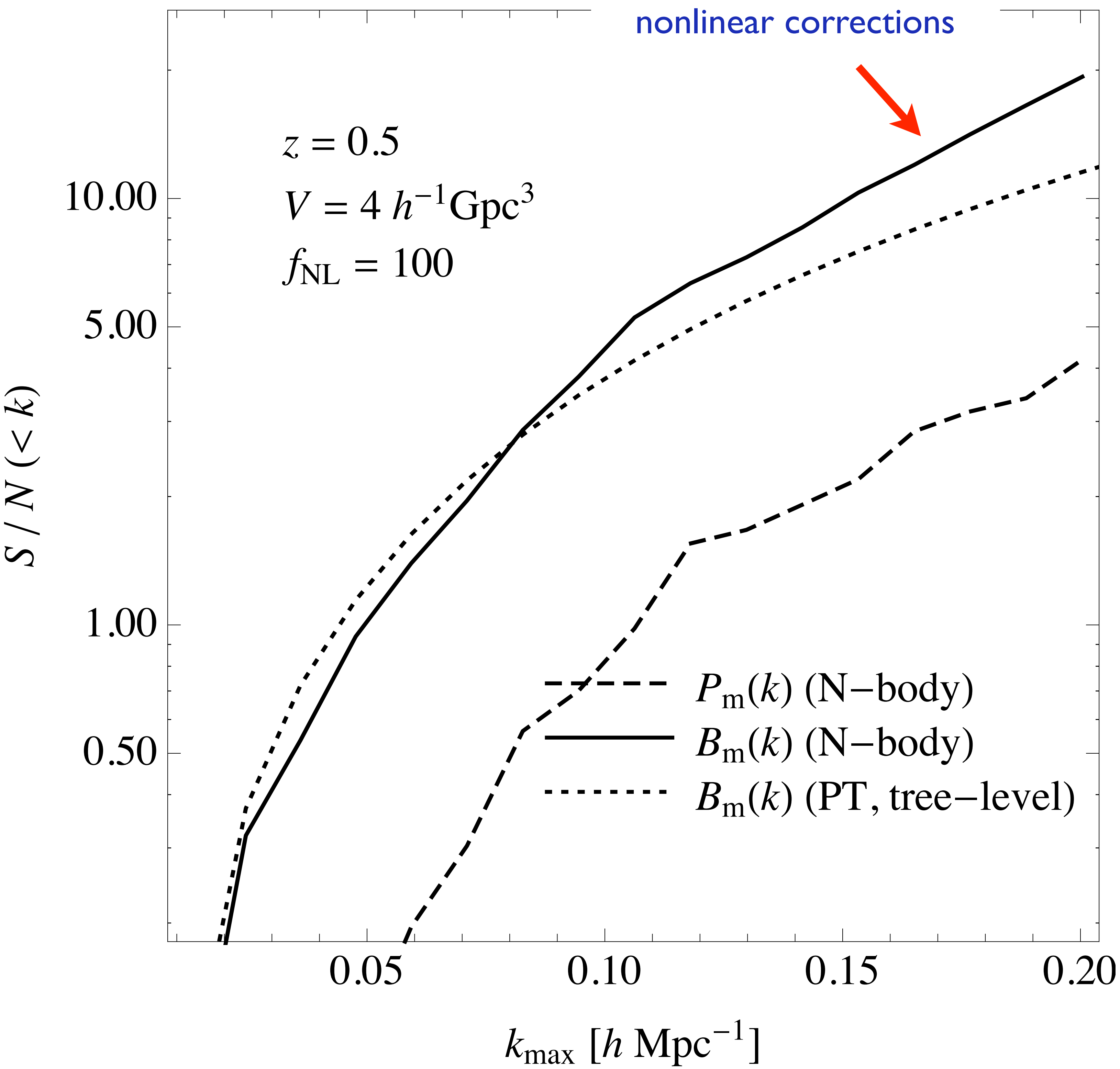




Matter Power Spectrum vs Matter Bispectrum



Cumulative signal-to-noise for the effect of NG initial conditions.

Sum of all configurations up to k_{max}

$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{max}} \frac{(P_{NG} - P_G)^2}{\Delta P^2}$$

$$\left(\frac{S}{N}\right)^2_B = \sum_{k_1,k_2,k_3}^{k_{max}} \frac{(B_{NG} - B_G)^2}{\Delta B^2}$$

ES, Crocce & Desjacques
(*in preparation*)

Effects of PNG on the **galaxy power spectrum**

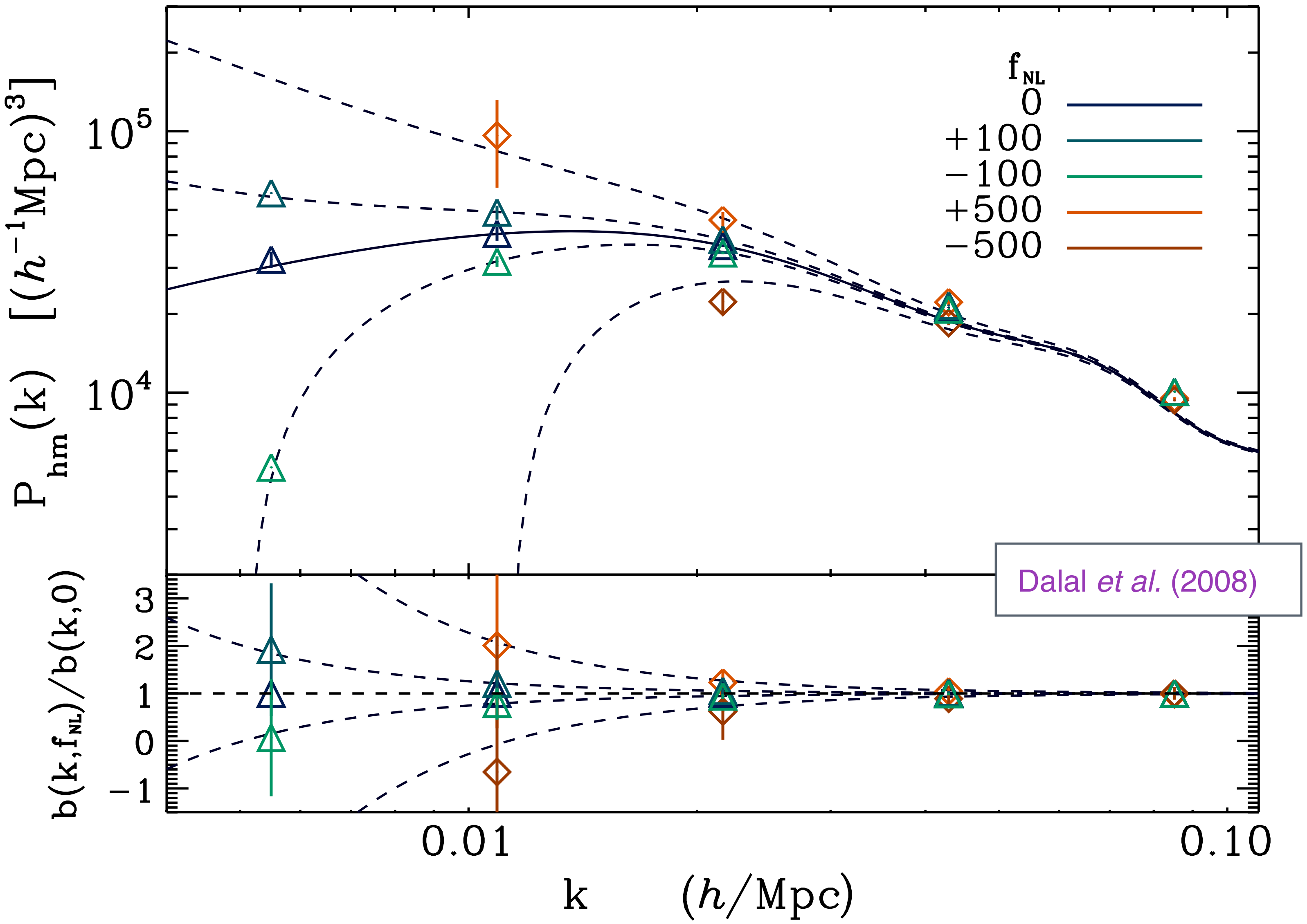
Dalal *et al.* (2008):

$$\delta_g(\vec{k}) = [b_1 + \Delta b_1(f_{NL}, k)]\, \delta_{\vec{k}} + \ldots \;\rightarrow\; P_g(k) = [b_1 + \Delta b_1(f_{NL}, k)]^2\, P(k)$$

"Gaussian" bias

Scale-dependent correction due to local non-Gaussianity

$$\Delta b_{1,NG}(f_{NL}, k) \sim \frac{f_{NL}}{D(z)\, k^2}$$



Effects of PNG on the **galaxy bispectrum**

Clearly, the effect on galaxy bias affects as well the **galaxy bispectrum**

$$B_g(k_1,k_2,k_3) = b_1^3\,B(k_1,k_2,k_3) + b_1^2\,b_2\,P(k_1)\,P(k_2) + 2\ \mathrm{perm.} + \ldots$$

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$b_{1,G} + \Delta b_{1,NG}(f_{NL}, k)$ $\quad$ $b_{2,G} + \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2)$

Scale-dependent bias corrections

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$$\Delta b_{1,NG}(f_{NL}, \vec{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sd}(f_{NL}, b_{1,G}, \vec{k})$$

$$\Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2) = \Delta b_{2,si}(f_{NL}) + \Delta b_{2,sd}(f_{NL}, b_{1,G}, b_{2,G}, \vec{k}_1, \vec{k}_2)$$

Giannantonio & Porciani (2010)
Baldauf, Seljak & Senatore (2010)

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Primordial component
(*large scales*)

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Primordial component (*large scales*)

Effect on nonlinear evolution (*small scales*)

$$\Delta b_{1,NG}(f_{NL}, \vec{k}) = \Delta b_{1,si}(f_{NL}) + \Delta b_{1,sd}(f_{NL}, b_{1,G}, \vec{k})$$

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Giannantonio & Porciani (2010)
Baldauf, Seljak & Senatore (2010)

- We test this model in N-body simulations with local NG initial conditions
- We fit ***all*** triangular configurations up to k = 0.07 h/Mpc for $b_{1,G}$, $b_{2,G}$, $\Delta b_{1,G}$ and $\Delta b_{2,G}$

$$\langle \delta\delta\delta_h \rangle = \delta_D(\vec{k}_{123})\, B_{mmh}$$

$$\langle \delta_h\delta_h\delta_h \rangle = \delta_D(\vec{k}_{123})\, B_h$$

$$P_h \;\rightarrow\; b_{1,G},\; \Delta b_{1,si}$$

$$B_{h,G} \;\rightarrow\; b_{2,G}$$

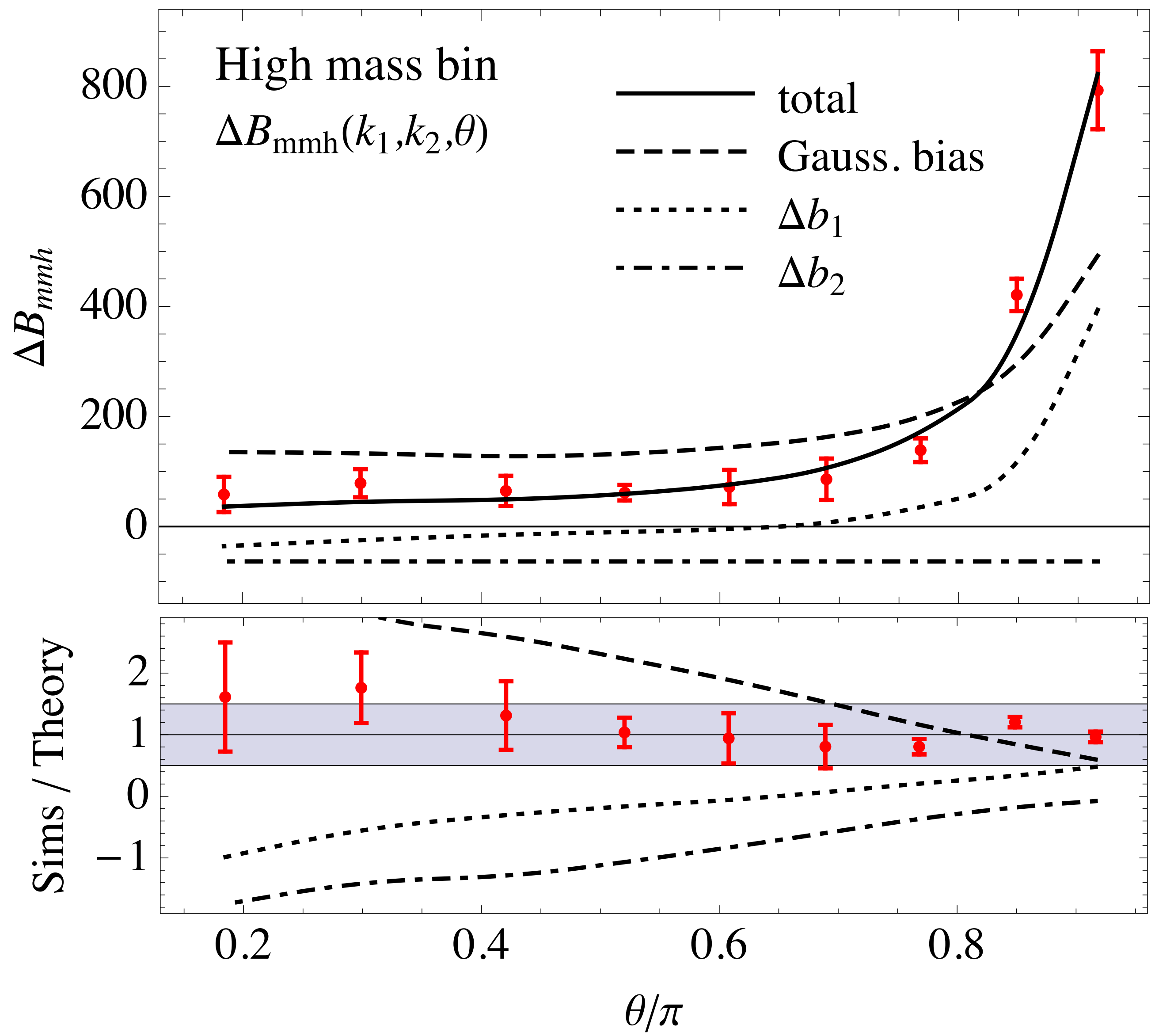
$$\Delta B_{h,NG} \;\rightarrow\; \Delta\, b_{2,si}$$

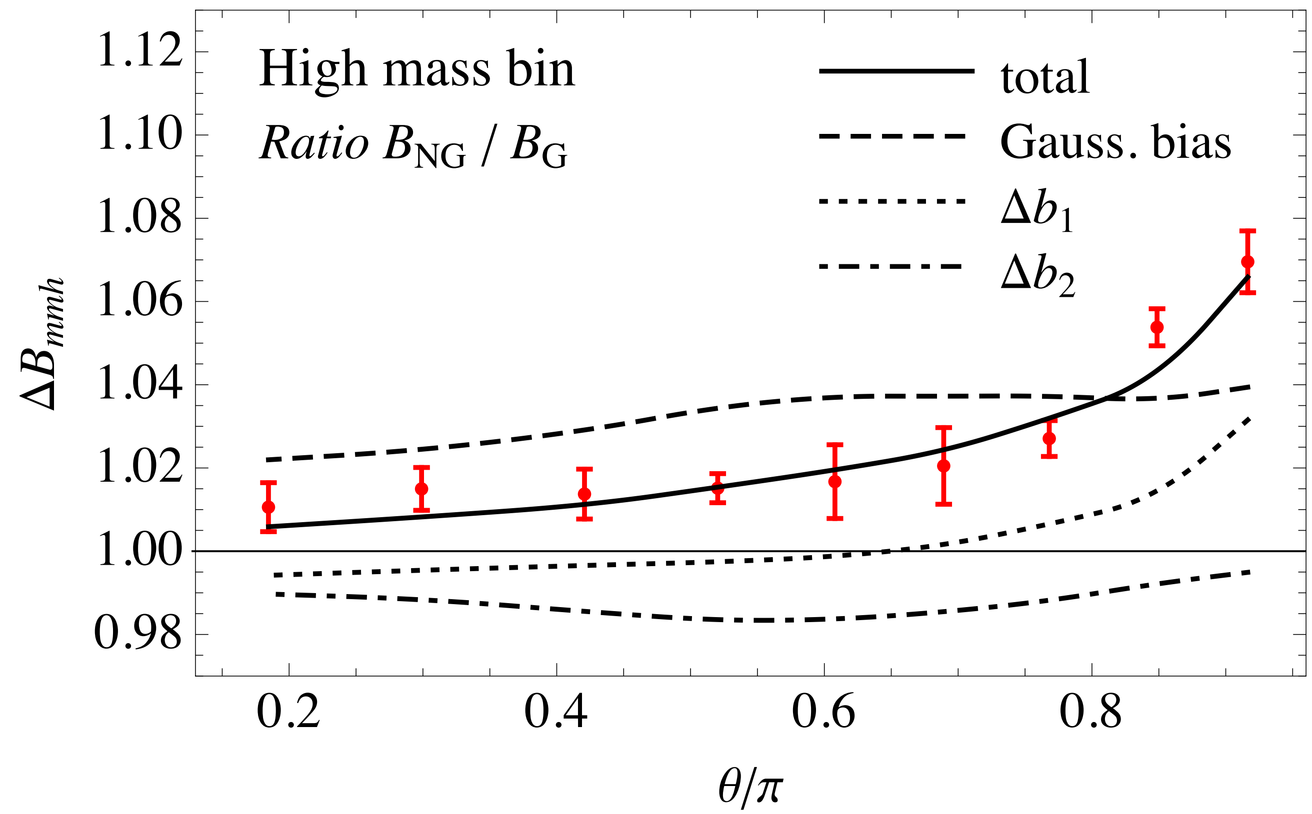
Effects of PNG on the **galaxy bispectrum**

Matter-matter-halo bispectrum:

$$B_{mmh}(k_1, k_2; k_3) = b_1(f_{NL}, k)\, B(k_1, k_2, k_3) + b_2(f_{NL}, k_1, k_2)\, P(k_1)\, P(k_2)$$

Generic configurations $B(k_1, k_2, \theta)$ as a function of θ with $k_1 = 0.1\ h/\mathrm{Mpc}$, $k_2 = 1.5\ k_1$





ES, Crocce & Desjacques (*in preparation*)

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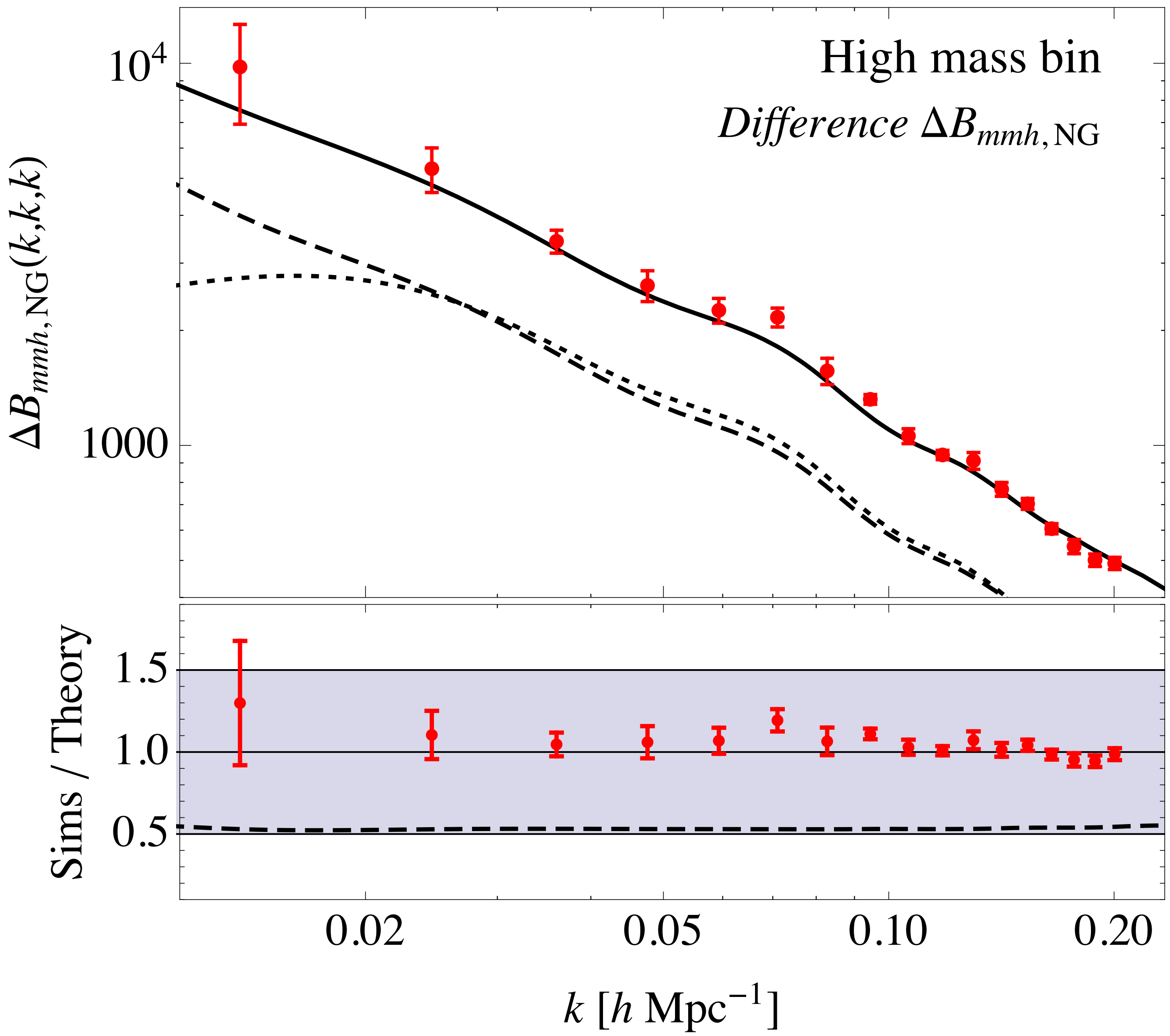
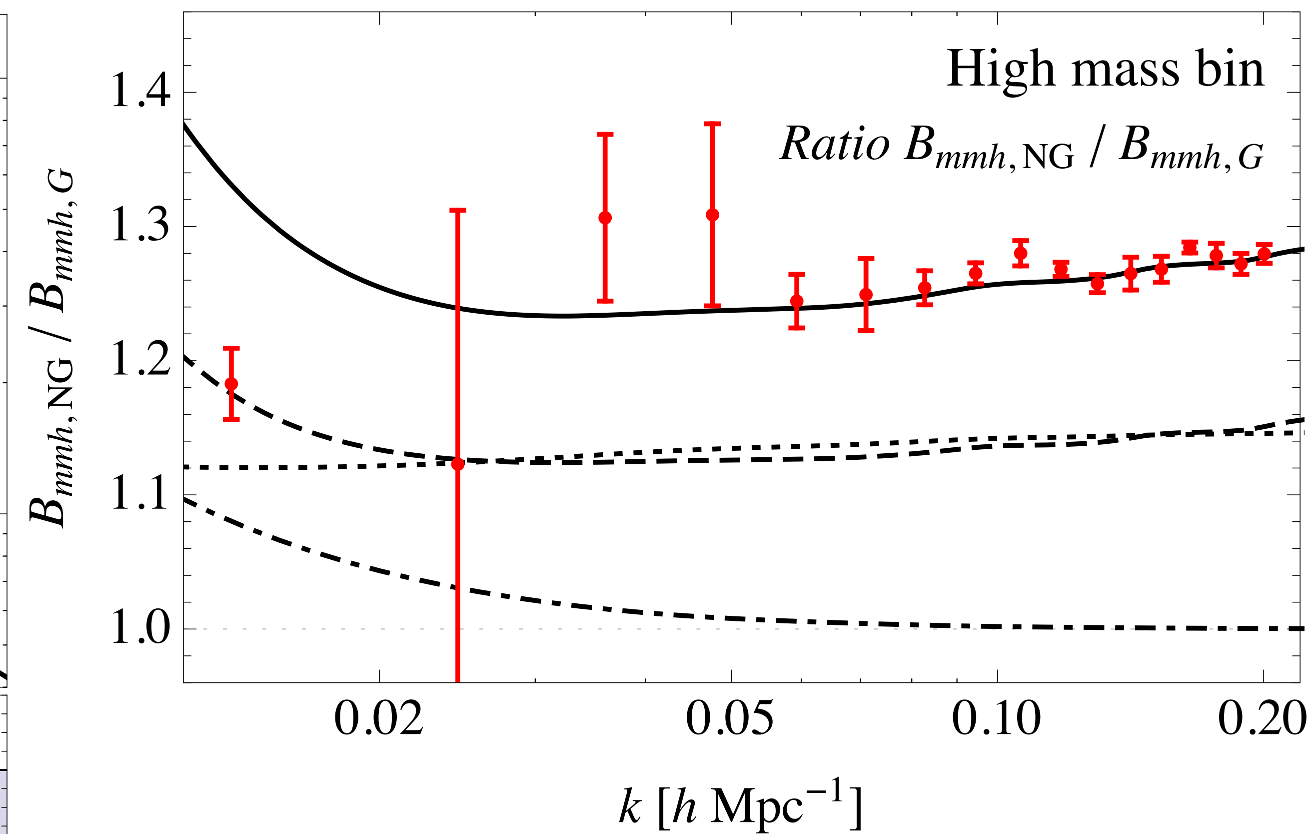
Squeezed configurations B(Δk, k, k)
as a function of k with Δk = 0.01 h/Mpc





ES, Crocce & Desjacques (*in preparation)*

Effects of PNG on the **galaxy bispectrum**

Halo bispectrum:

$$B_h(k_1,k_2,k_3) = b_1^3(f_{NL},k)\,B(k_1,k_2,k_3)$$
$$+b_1(f_{NL},k_1)b_1(f_{NL},k_2)b_2(f_{NL},k_1,k_2)\,P(k_1)\,P(k_2) + cyc.$$

Squeezed configurations B(Δk, k, k)
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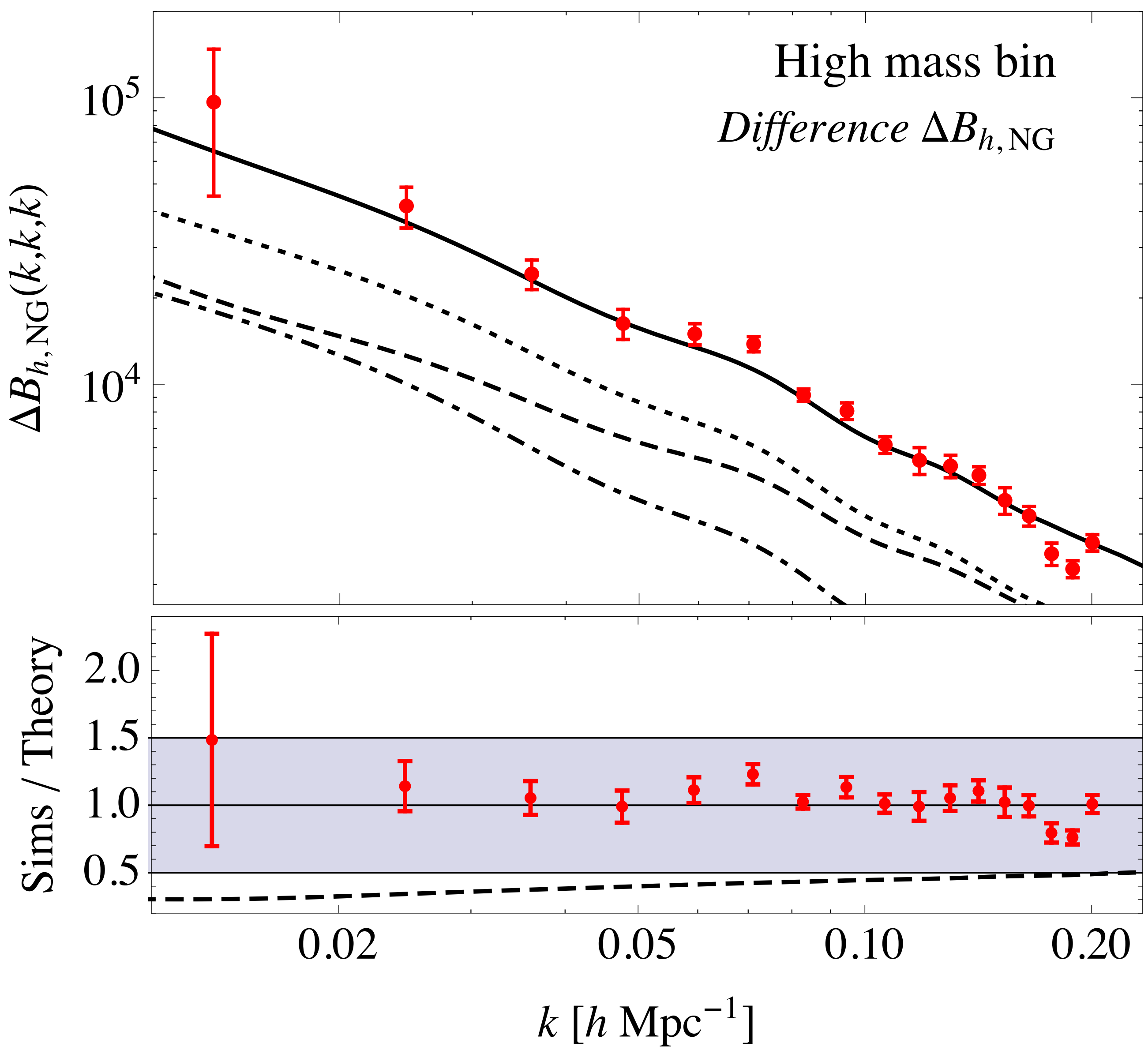

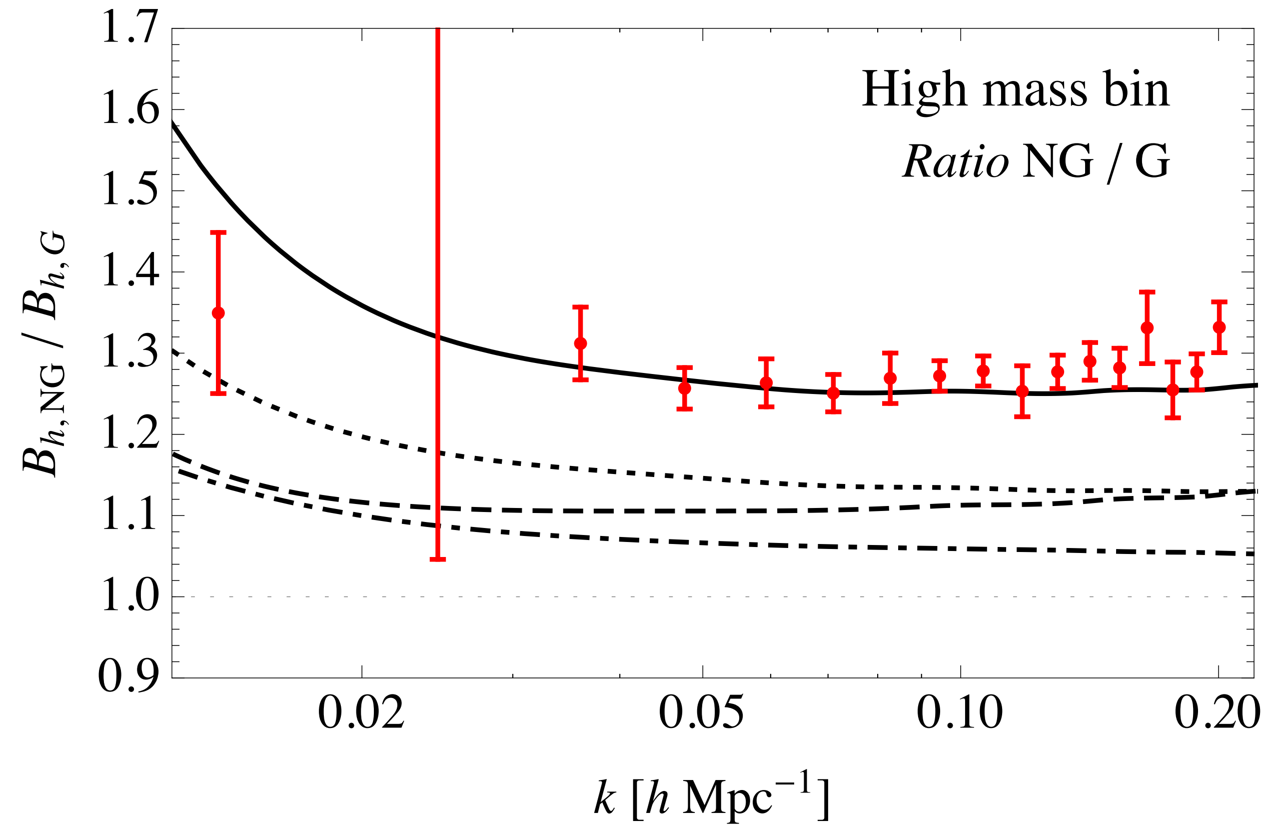

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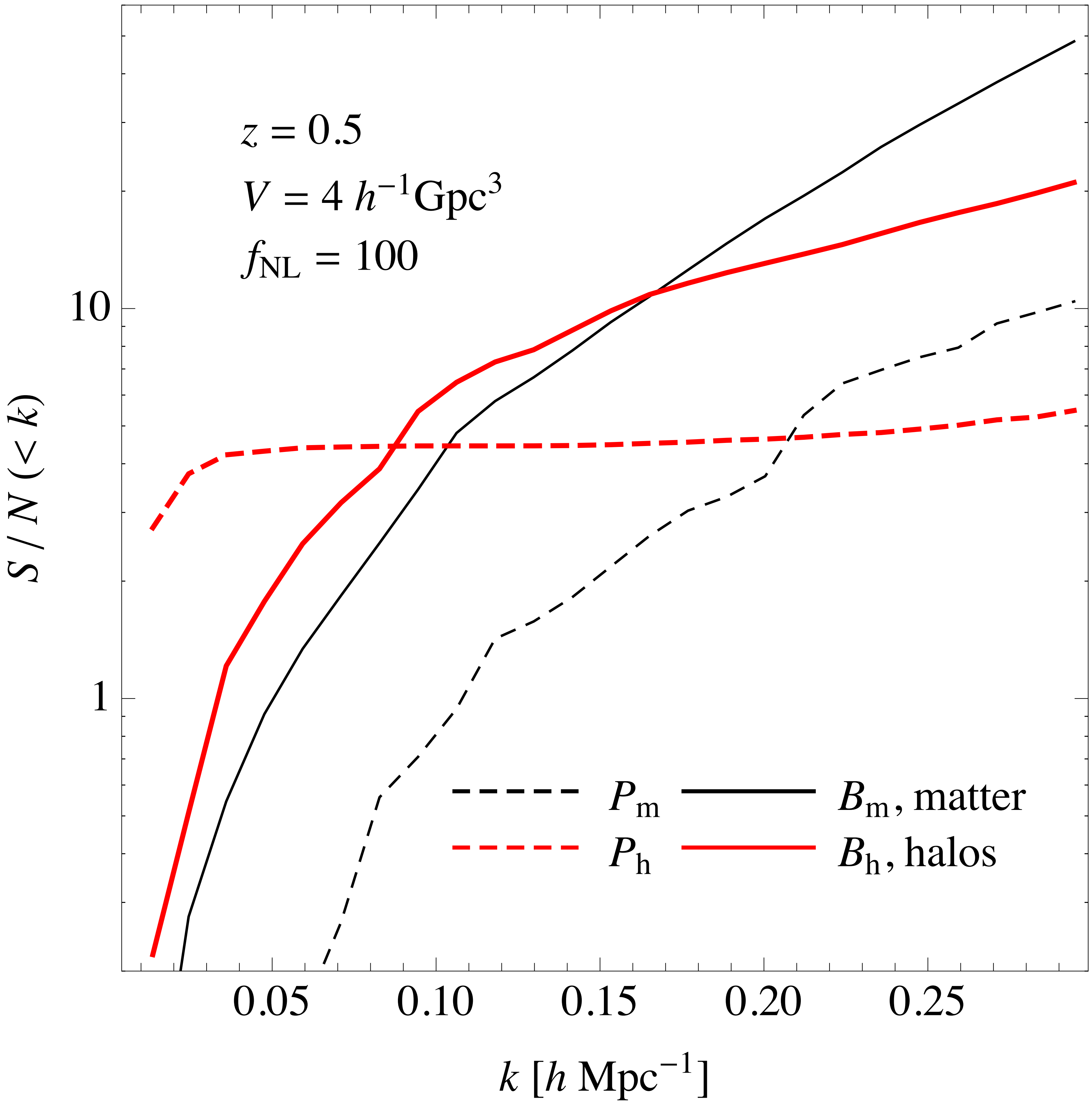
Power Spectrum vs. Bispectrum



Cumulative signal-to-noise for the effect of NG initial conditions on matter and galaxy correlators (P & B)

Sum of all configurations up to k_{max}

$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{max}} \frac{(P_{NG} - P_G)^2}{\Delta P^2}$$

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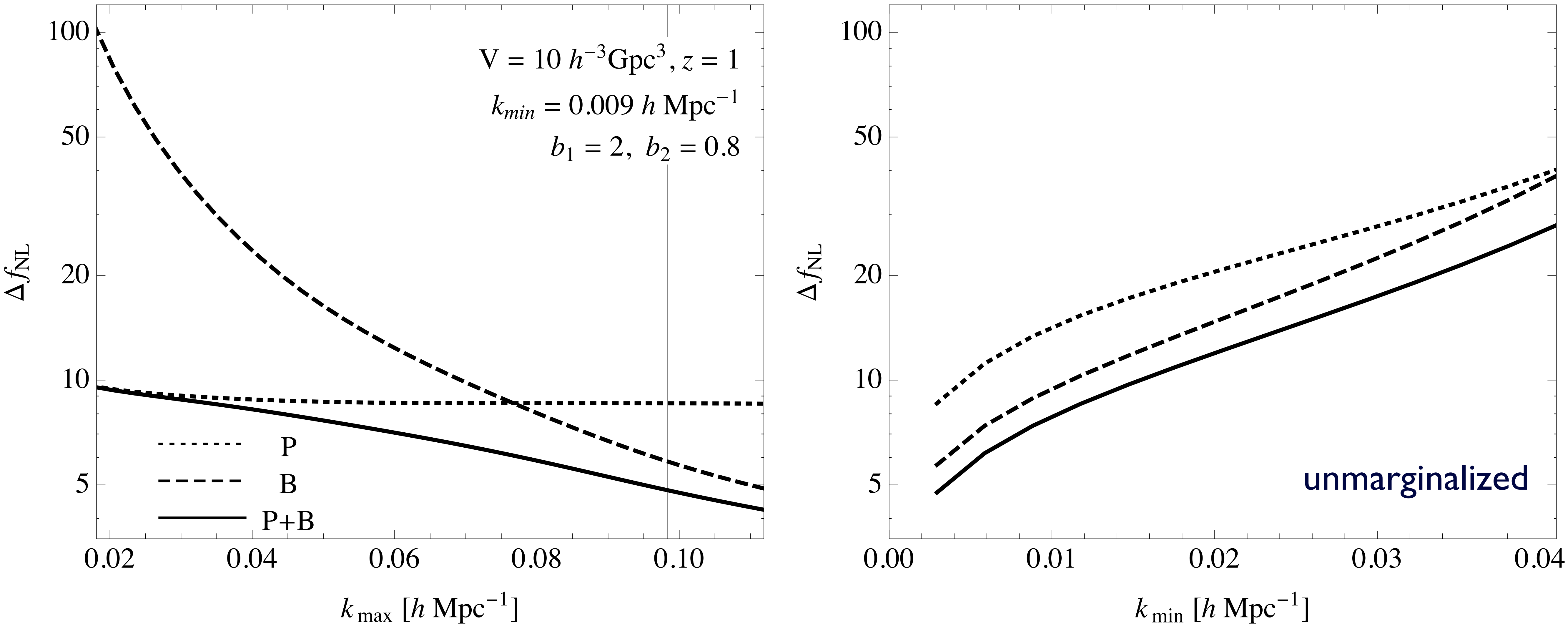
An *un*realistic Fisher matrix analysis

Assuming *perfect knowledge* of a complete galaxy population in a 10 Gpc3 volume at redshift z with density 10^-3 Mpc^-3

Ignoring *any* complication no matter how relevant and pertinent (covariance, redshift distorsions, selection function, degeneracies, etc ...)

We can estimate the uncertainty on fNL (local) from Power Spectrum & Bispectrum (& both)



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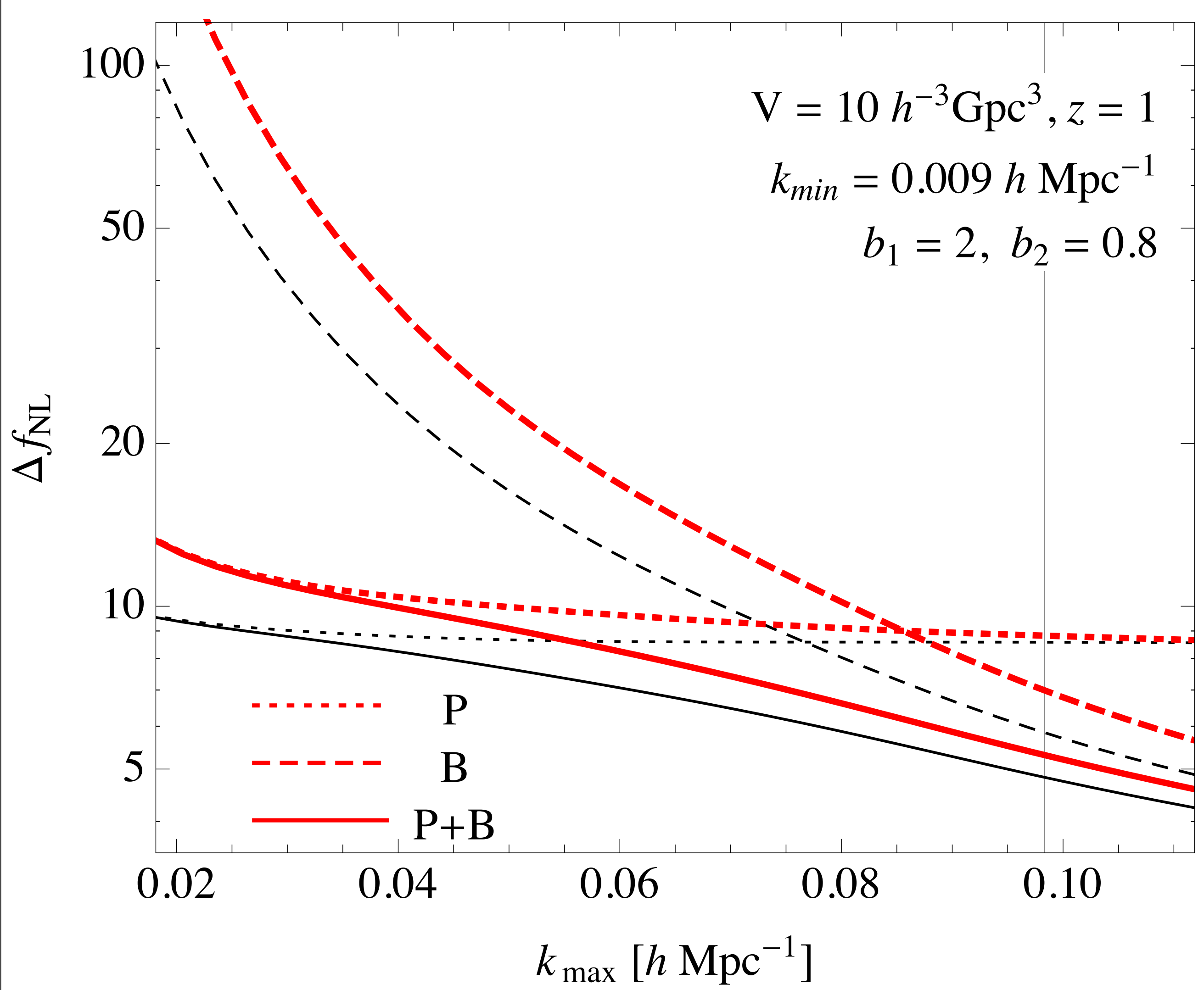

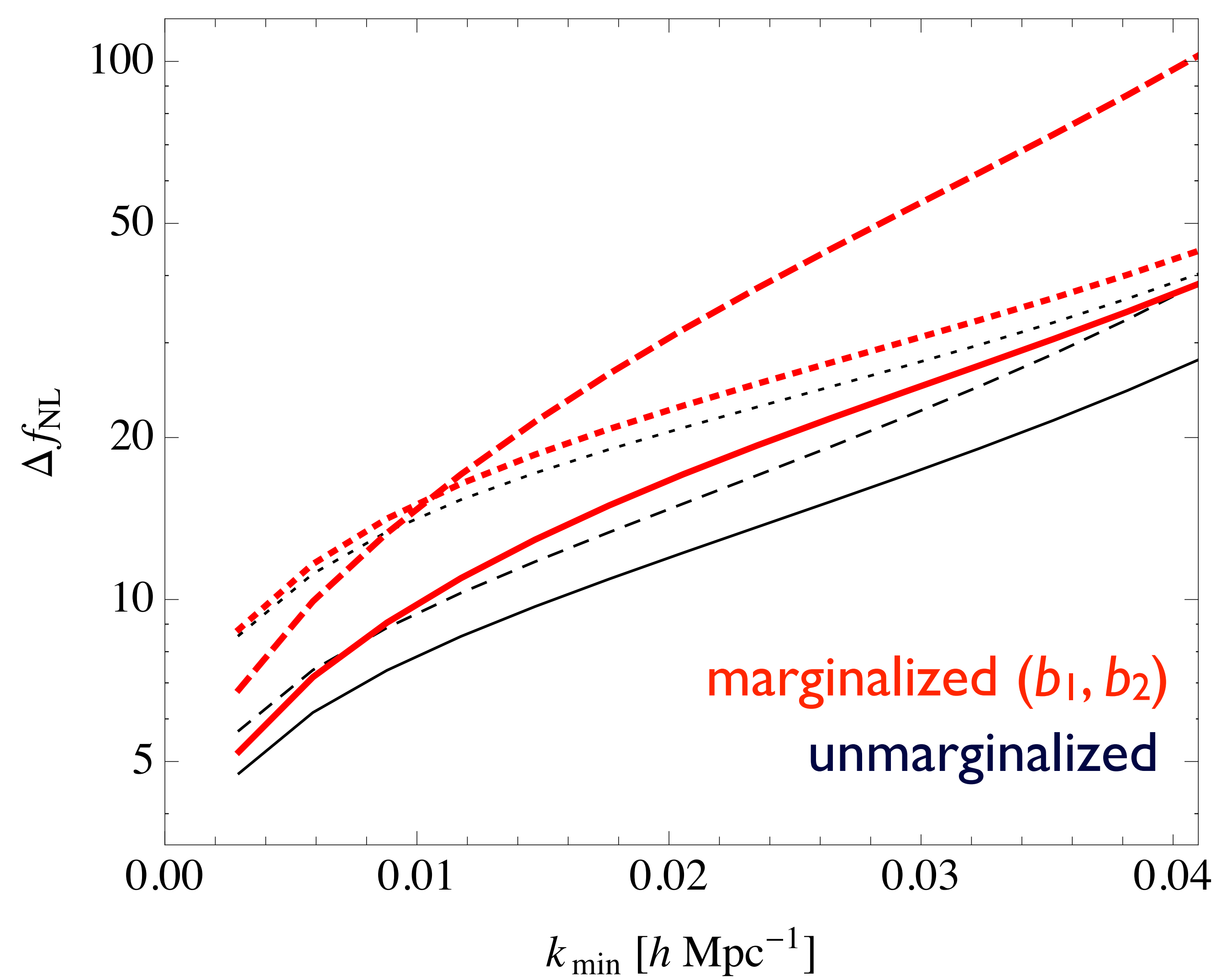

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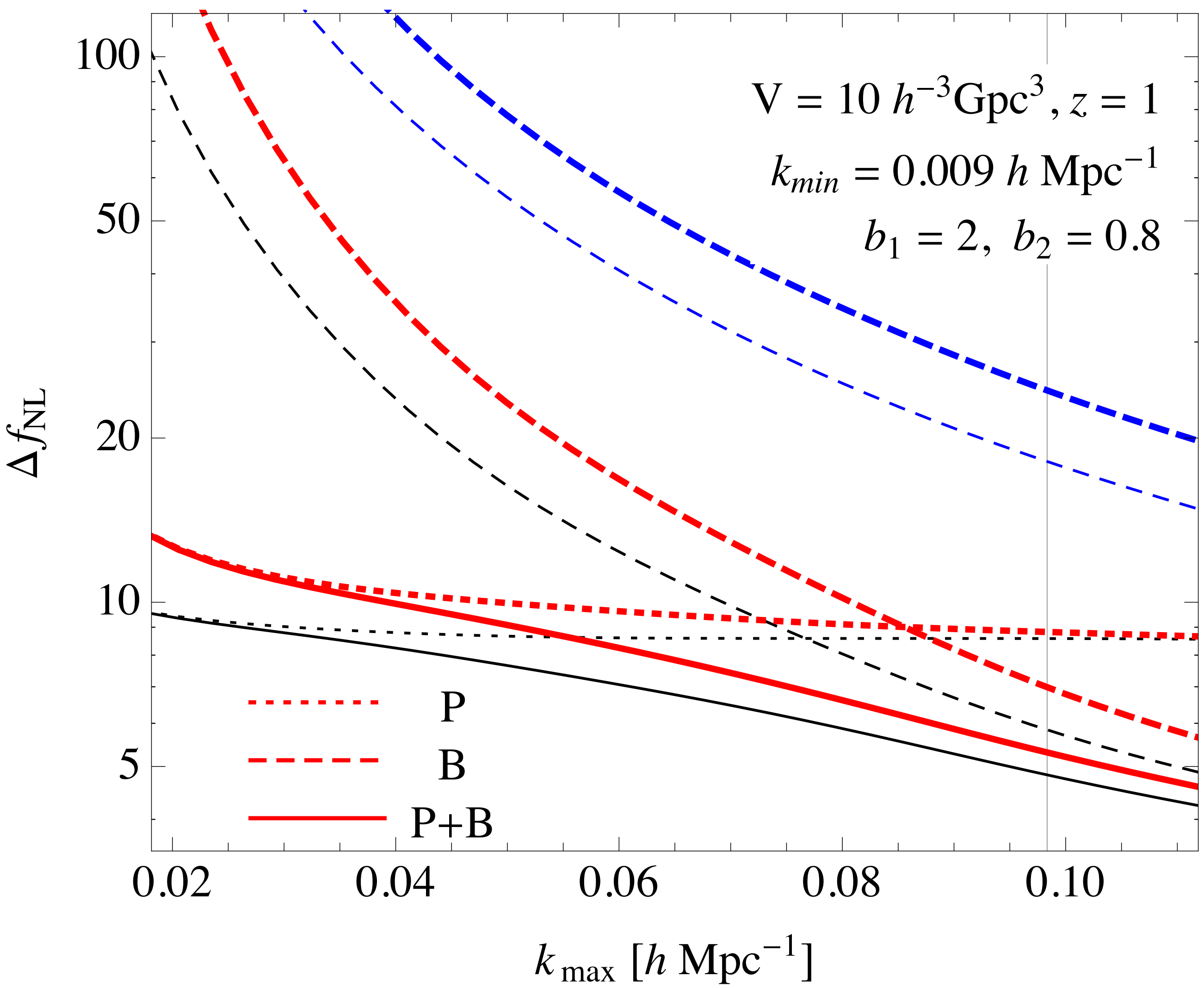
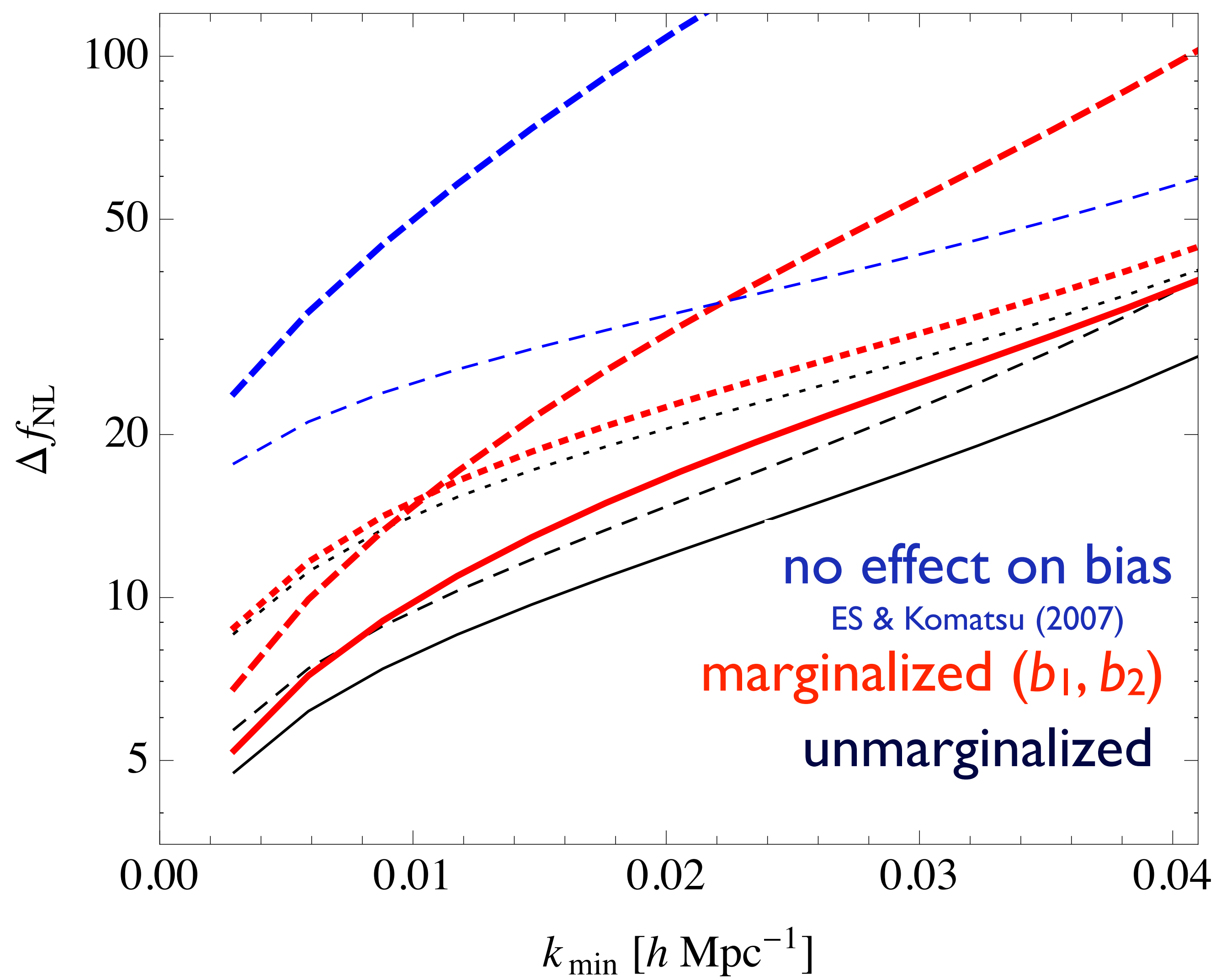
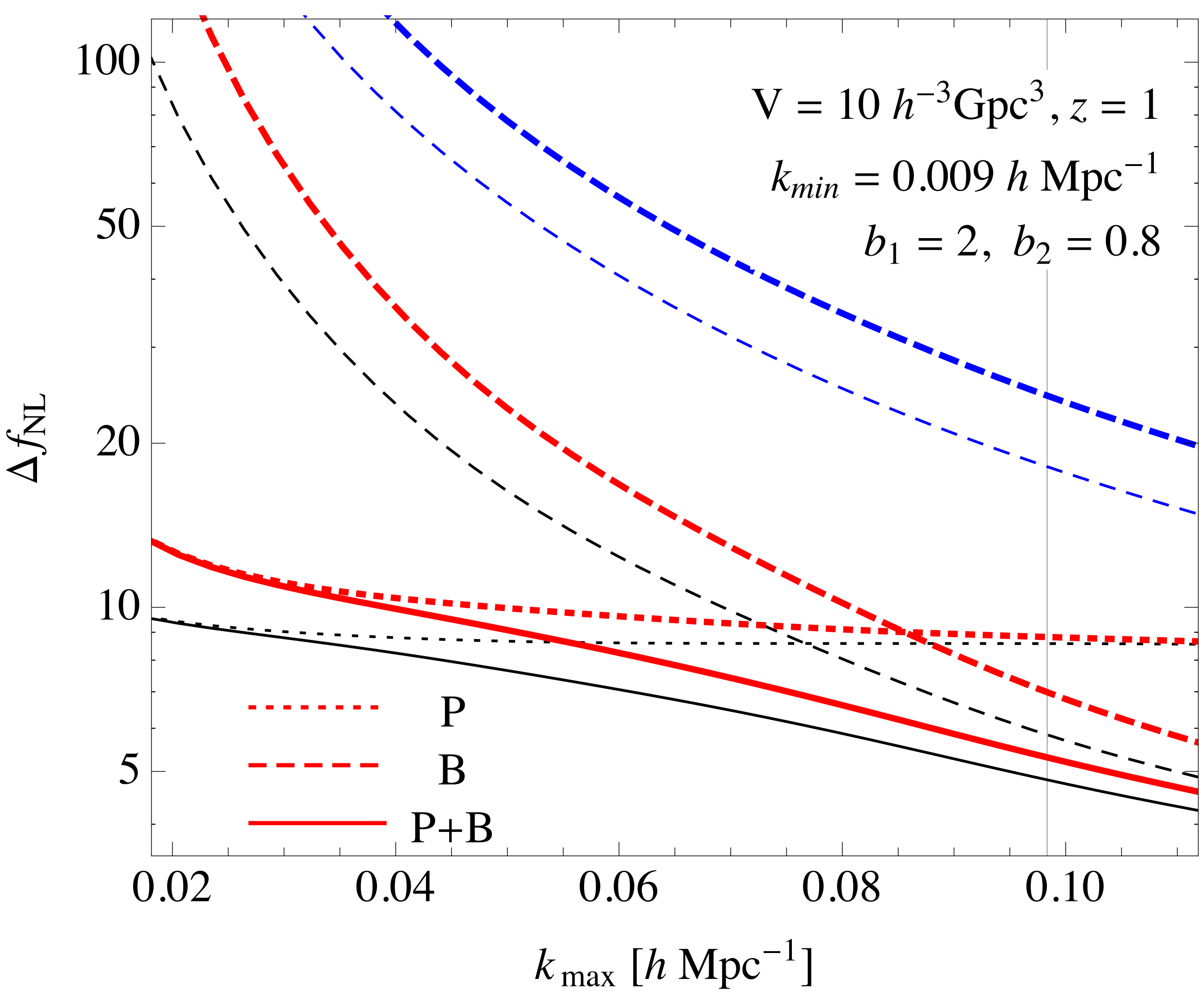
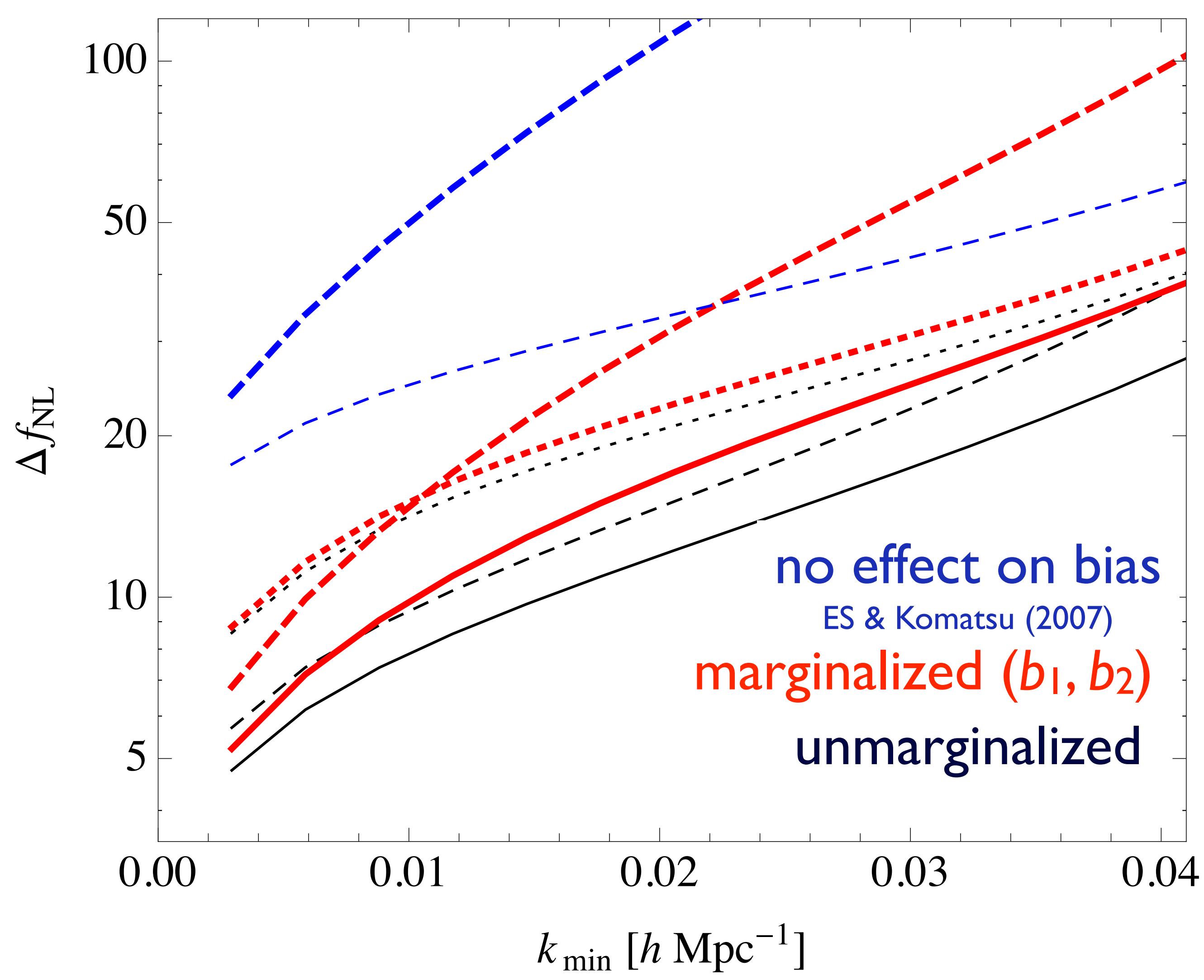




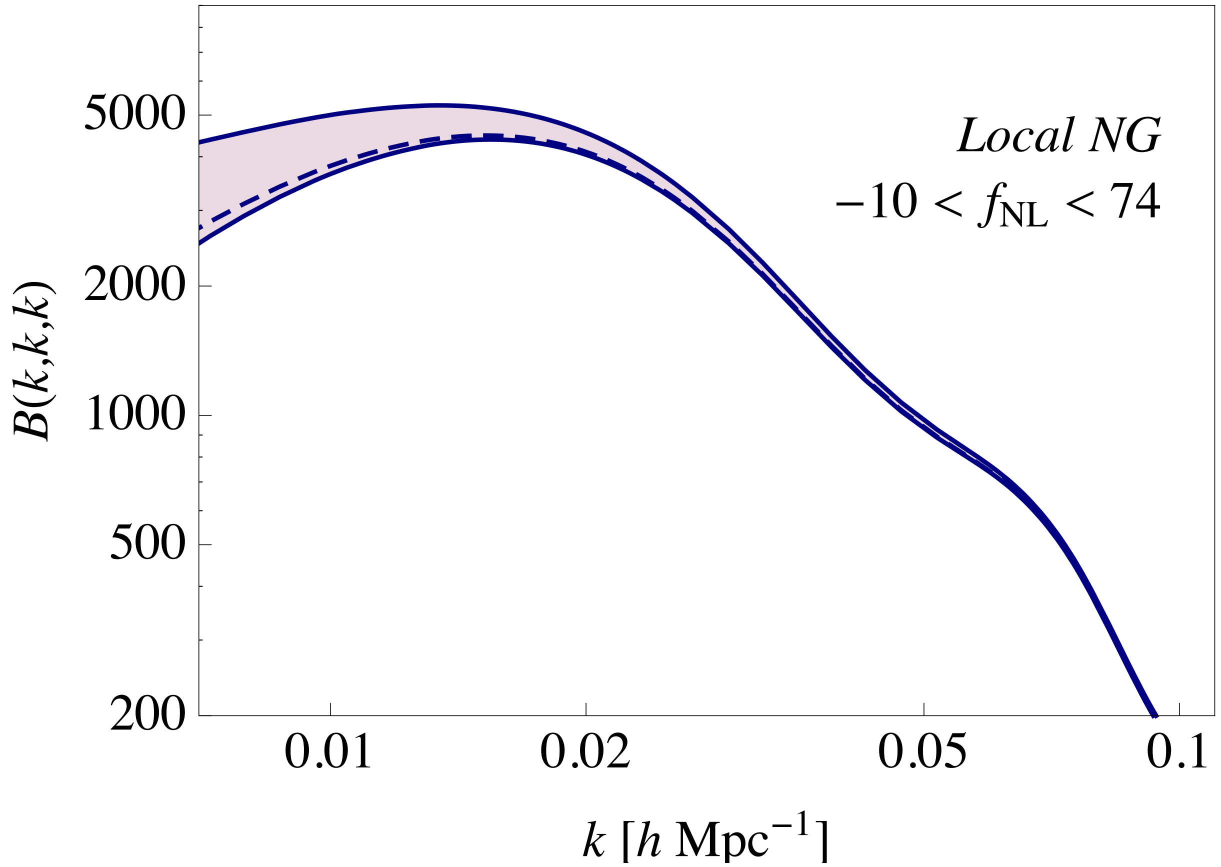
Conclusions

- We have a (relatively simple) model for the large scales galaxy bispectrum with local NG initial conditions
- Bispectrum measurements in LSS surveys can confirm and improve constraints on f_{NL} from the power spectrum (particularly for non-local models ...)



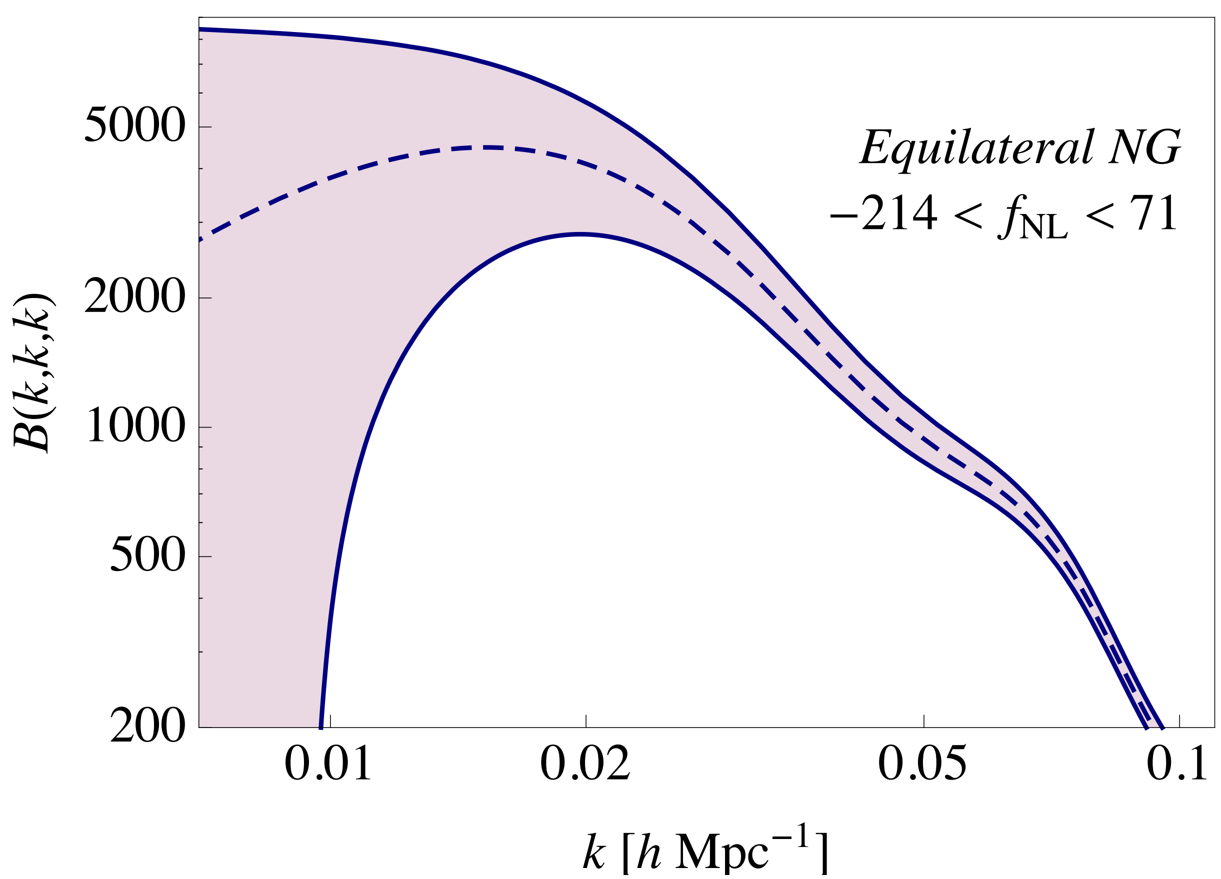


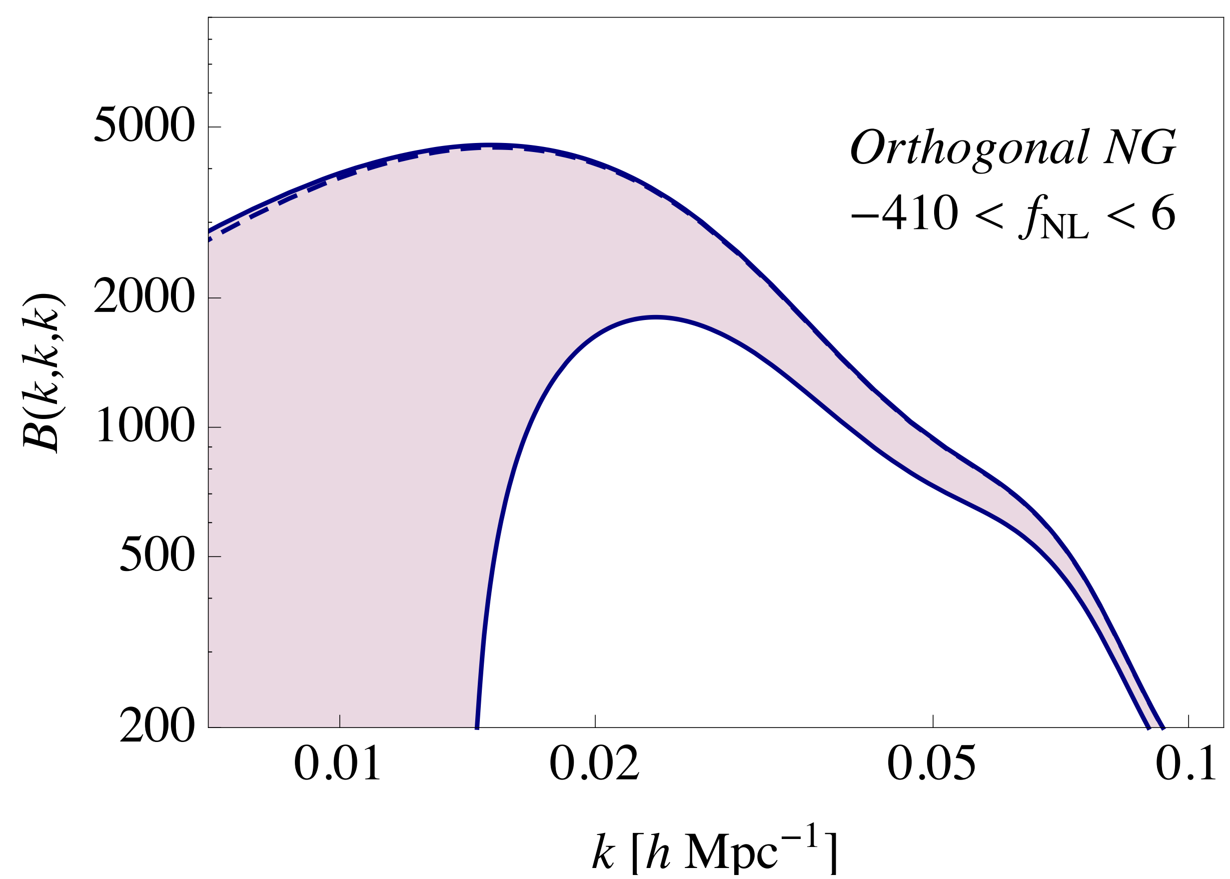
The matter bispectrum and PNG: *large scales*



Current CMB constraints for different models of non-Gaussianity as uncertainties on the equilateral configurations of the matter bispectrum

$$B \simeq B_0 + B_G^{tree}[P_0]$$



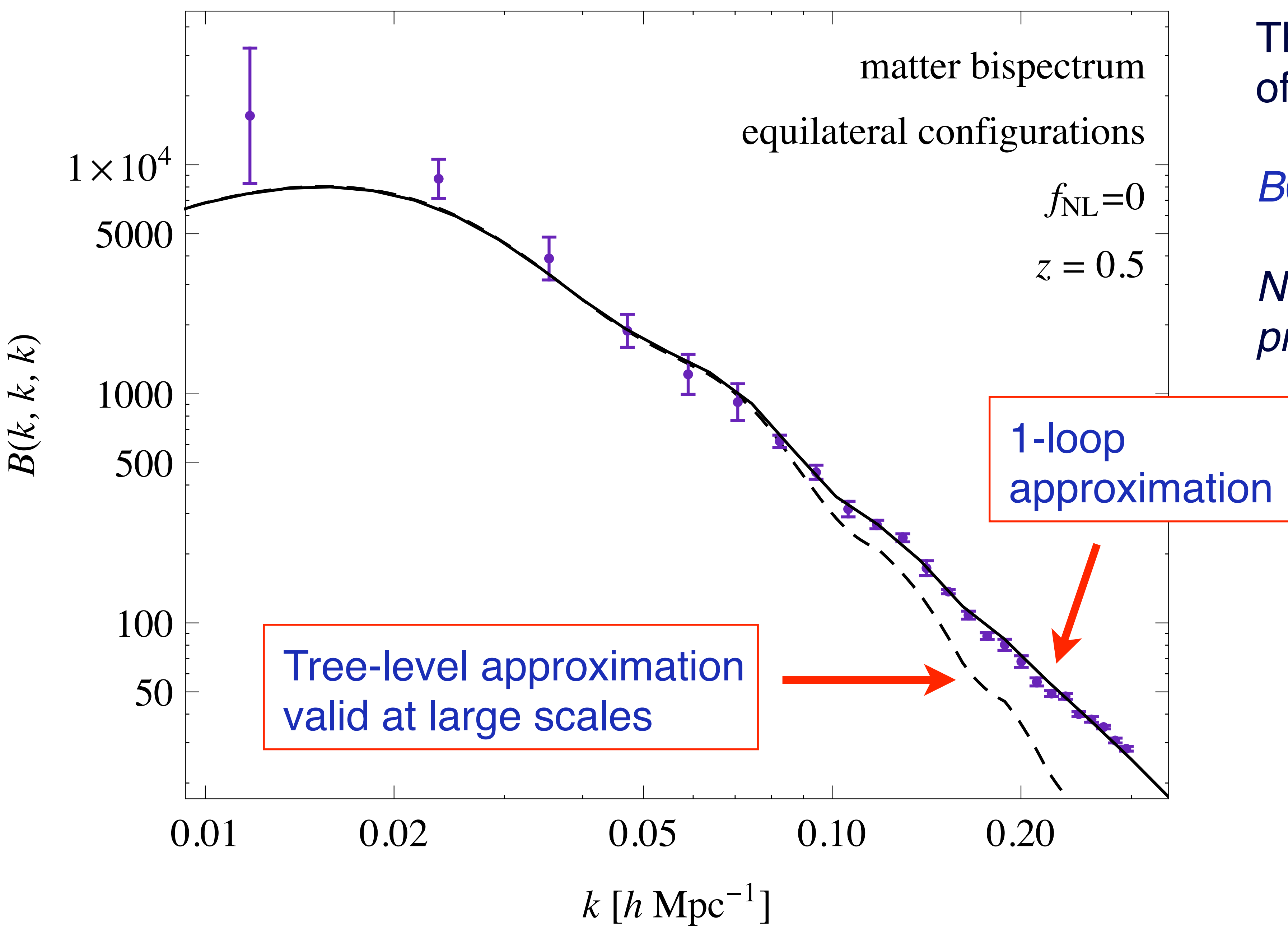


The **Matter Bispectrum** induced by **Gravity**

$$B_G = B_G^{tree}[P_0] + B_G^{loop}[P_0]$$

$$B_G^{tree}(k_1, k_2, k_3) = 2\, F_2(\vec{k}_1, \vec{k}_2)\, P_0(k_1)\, P_0(k_2) + 2 \text{ perm.}$$

The bispectrum induced by gravity has a well defined dependence on *scale* and on the *shape*



The equilateral configurations of the matter bispectrum:

B(*k*, *k*, *k*) vs. *k*

Numerical simulations and PT predictions

E.S., M. Crocce, & V. Desjacques (2010)

Non-Gaussianity from **Gravitational Instability**

At large scales fluctuations are small, $\sigma_\delta \ll 1$, even at low redshift
we can study their evolution in terms of **Perturbation Theory**

Equations of motion for matter density and velocity:

$$\delta, \;\; \vec{v}$$

- Continuity eq. $$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\vec{v}] = 0$$
- Euler eq. $$\frac{\partial \vec{v}}{\partial \tau} + +\mathcal{H}\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}\phi$$
- Poisson eq. $$\nabla^2 \phi = \frac{3}{2}\mathcal{H}^2 \Omega_m \delta$$

Perturbative solution for the matter density, in Fourier space

$$\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}} + \delta^{(2)}_{\vec{k}} + \ldots$$

Linear solution

$$\delta^{(2)}_{\vec{k}} = \int d^3q \, F_2(\vec{k}-\vec{q},\vec{q}) \, \delta^{(1)}_{\vec{k}-\vec{q}} \delta^{(1)}_{\vec{q}}$$

Quadratic nonlinear correction

Initial conditions

$$\langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1+\vec{k}_2) P_0(k_1)$$

$$\langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \delta^{(1)}_{\vec{k}_3} \rangle = \delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3) \, B_0(k_1,k_2,k_3)$$

$$\langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \delta^{(1)}_{\vec{k}_3} \delta^{(1)}_{\vec{k}_4} \rangle = \delta_D(\vec{k}_1+\ldots+\vec{k}_4) \, T_0(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)$$

B_0 and T_0 vanish for Gaussian initial conditions!

Perturbative solution for the matter 3-point function

$$\langle \delta\delta\delta \rangle = \langle \delta^{(1)}\delta^{(1)}\delta^{(1)} \rangle + \langle \delta^{(1)}\delta^{(1)}\delta^{(2)} \rangle + \ldots$$

loop corrections

$= B_0 = 0$ for Gaussian initial conditions

non-zero bispectrum induced by gravity!

Non-Gaussianity from **Galaxy Bias** (more problems?)

Additional non-Gaussianity in the **galaxy distribution** is induced by **nonlinear galaxy bias**

*The relation between the **observed galaxy overdensity** and the matter density is nonlinear*

$$\delta_g(x) \equiv \frac{n_g(x) - \bar{n}_g}{\bar{n}_g} = f\,[\delta(x)]$$

local bias

At large scales, we expand it in a Taylor series

$$\delta_g(x) = b_1\,\delta(x) + \frac{1}{2}\,b_2\,\delta^2(x) + \ldots$$

Linear bias

Quadratic bias correction

*Perturbative solution for the **galaxy** 3-point function*

$$\langle \delta_g \delta_g \delta_g \rangle = b_1^3\,\langle \delta\delta\delta \rangle + b_1^2\,b_2\,\langle \delta\delta\delta^2 \rangle + \ldots$$

matter bispectrum

bispectrum induced by nonlinear bias

$$B_g(k_1, k_2, k_3) = b_1^3\,B(k_1, k_2, k_3) + b_1^2\,b_2\,P(k_1)\,P(k_2) + 2\ \text{perm.} + \ldots$$

The component induced by bias has a *different dependence* on the *shape of the triangle*

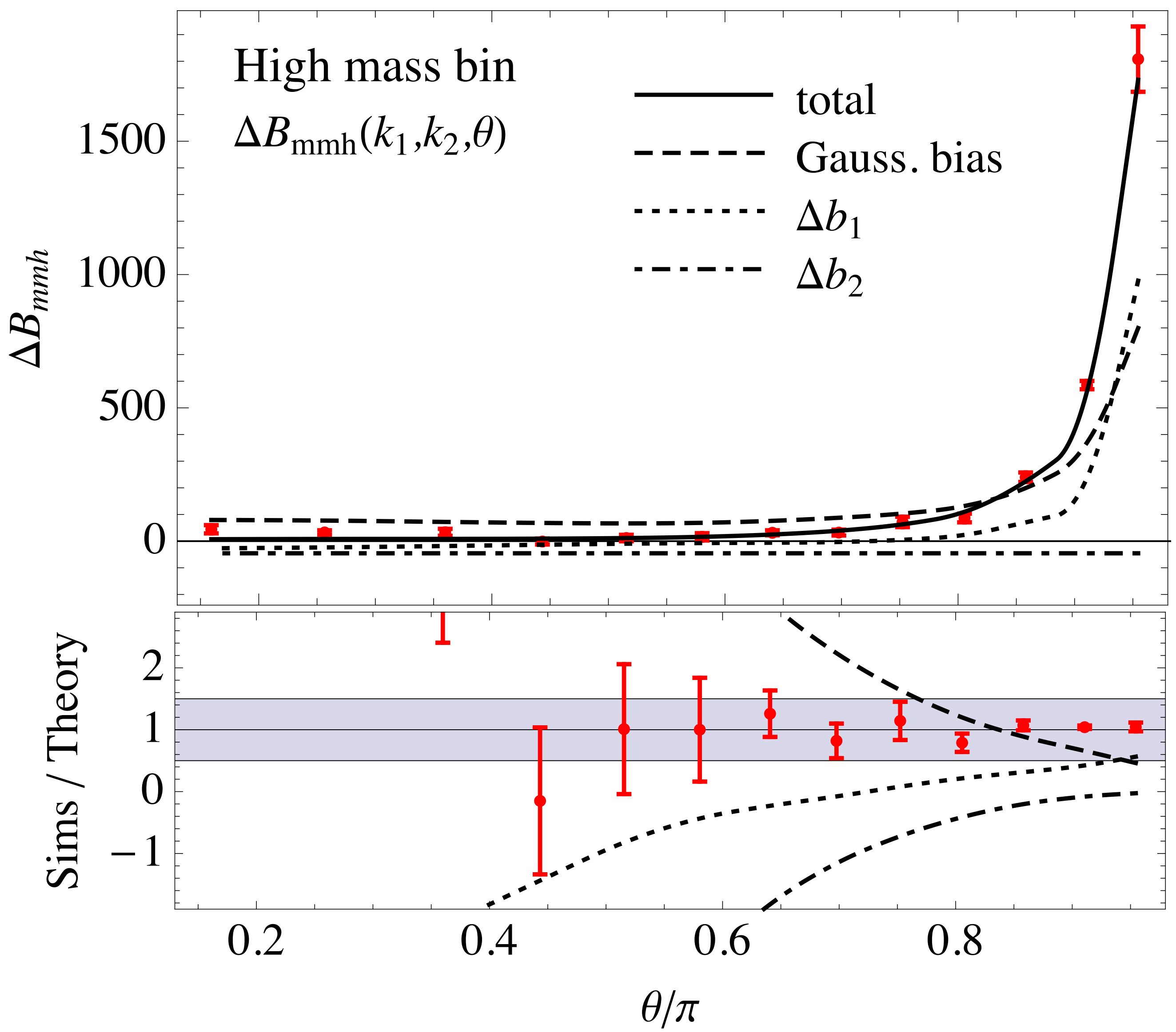
Effects of PNG on the **galaxy bispectrum**

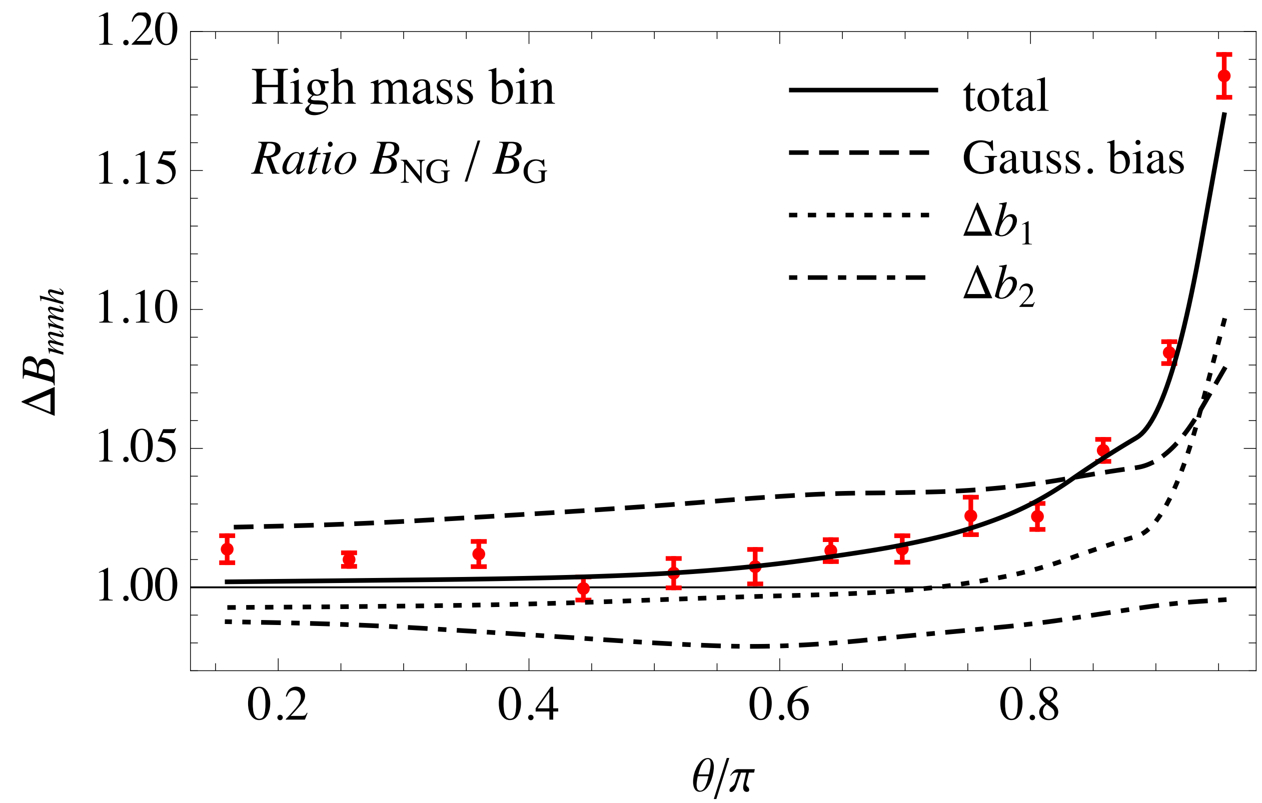
Clearly, the effect on galaxy bias affects as well the **galaxy bispectrum**

$$b_{1,G} + \Delta b_{1,NG}(f_{NL}, k) \qquad b_{2,G} + \Delta b_{2,NG}(f_{NL}, \vec{k}_1, \vec{k}_2)$$

Scale-dependent bias corrections

$$B_g(k_1, k_2, k_3) = b_1^3\, B(k_1, k_2, k_3) + b_1^2\, b_2\, P(k_1)\, P(k_2) + 2 \text{ perm.} + ...$$





ES, Crocce & Desjacques (*in preparation)*

The matter bispectrum and PNG: *small scales*